UNIVERSITÄT ZU LÜBECK
INSTITUT FÜR MATHEMATIK

Oberseminar des Instituts für Mathematik

Wir laden ein zu dem Vortrag

Complexity Analysis of Quantizations of Multidimensional Stochastic Differential Equations

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Let $s \in [1,\infty)$, let $(\Omega,\mathcal{A},P)$ be a probability space, and let $X:\Omega\to B$ be a random element with values in a separable $\mathbb{R}$-Banach space $(B,\|\cdot\|_B)$ satisfying $\mathbb{E}\left[\|X\|_B^s\right]<\infty$. The *quantization problem for* X consists in approximating X by a B-valued random element $\widetilde{X}$ defined on $(\Omega,\mathcal{A},P)$ which takes only finitely many values. For any such $\widetilde{X}$ the *quantization error* is defined by $e^{(s)}(X,\widetilde{X},B) := \left(\mathbb{E}\left[\|X-\widetilde{X}\|_B^s\right]\right)^{1/s}$, and, for $N\in\mathbb{N}$, we call $\widetilde{X}$ an N*-quantization* of X if $|\widetilde{X}(\Omega)|\le N$. Moreover, the N*th minimal quantization error*

$$e_N^{(s)}(X,B) := \inf\left\{ \left(\mathbb{E}\left[\|X-\widetilde{X}\|_B^s\right]\right)^{1/s} \,\middle|\, \widetilde{X} \text{ is an } N\text{-quantization of } X\right\}$$

is the minimal error that can be achieved by any N-quantization of X. In this talk we focus on the quantization problem for X being the solution process of a d-dimensional SDE, and we consider the spaces $B_p := L_p([0,1];\mathbb{R}^d)$, $p\in[1,\infty)$, and $B_\infty := \mathcal{C}([0,1];\mathbb{R}^d)$ equipped with the L_p-norm and the supremum norm, respectively. Our main interest lies in the investigation of the asymptotic behavior of the Nth minimal quantization error of X as N tends to infinity, which incorporates the determination of both the sharp rate of convergence and explicit asymptotic constants. Especially explicit asymptotic constants are so far unknown in the context of multidimensional SDEs. Furthermore, as part of our analysis, for $p\in[1,\infty]$ we provide a method which yields a sequence $(\widetilde{X}_N^{(p)})_{N\in\mathbb{N}}$ of N-quantizations of X such that $\lim_{N\to\infty} \frac{e^{(s)}\left(X,\widetilde{X}_N^{(p)},B_p\right)}{e_N^{(s)}\left(X,B_p\right)} \le 1$. Such sequences of quantizations are called *strongly asymptotically optimal*.

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