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READING MATHEMATICAL TEXTS WITH PHILOSOPHICAL METHODS

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The paper presents a hermeneutic approach to teaching and learning mathematics in which the hermeneutic circle is translated to methods of reading mathematical texts with the help of the three decisive steps 1) making prior knowledge explicit, 2) interpreting texts, and 3) fusing horizons of prior understanding and text content.

INTRODUCTION

In their first year, many students of STEM disciplines encounter considerable difficulties in learning mathematics. These difficulties rarely reflect a lack of basic or specialized knowledge alone. Rather, the students lack strategies that would allow them to learn mathematics independently. It is well known that reading mathematical papers causes especially severe problems (Hilgert & Hoffmann & Panse, 2015). One reason for this is the economical style of presenting mathematical content in introductory textbooks, which abandons the context of discovery in favor of the context of justification. The goal of the present paper is to develop systematic methods for understanding mathematical texts. These methods support the independent reading of mathematical texts and so prepare students at an early stage for their future academic study of the subject.

Our educational concept combines specific mathematics-related writing, reading, and understanding-related tasks with meta-reflection. These methods help to question and analyze the appropriateness of the way in which mathematical content is typically presented. That is, our concept satisfies the requirement that mathematics, as almost all disciplines, engages in critical self-examination. Moreover, we expect that this method – an assumption that will have to be tested in the course of further research – will help students to make content intelligible both by means of enhancing their reading skills and by increasing their understanding how science (or mathematics) works. Specifically, we present hermeneutic methods for analyzing mathematical texts.

The basic idea is to develop philosophical methods of thinking and working into reading strategies for mathematical texts. A comprehensive understanding of mathematical texts can result from the interplay of five philosophical or "elementary methods of thinking" (Martens, 2003). These methods of thinking were derived by simplifying classical philosophical positions to their essential features for educational purposes (for more detail, see Schnieder, 2013). These five methods fostering comprehensive text understanding will be sketched in the following paragraphs.

Phenomenological methods: The objects, issues, or phenomena in the text are identified as specific examples and described precisely and in full detail. The large spectrum of different variations and modifications allows an unbiased (or less biased) perspective on theoretical explanations, interpretations and approaches through which the phenomena are treated in the text.

Analytical methods: These methods focus on the concepts and arguments in mathematical texts and their grammatical, semantic and logical structures. They detail intended meanings, justifications and ways of expressing content: What are the central claims in the text, how are they justified, and how are they presented formally and as regards content?

Speculative methods. Scientific insights and findings can be anticipated by thought experiments. Ideas and approaches for these experiments cannot be derived systematically, but finding them can be facilitated. Speculative methods foster structured thought experiments, for instance systematic variation of basic assumptions, premises, theorems end even whole theories. These methods also open the reader's eyes for ambiguities, contradictions and interruptions in the text.

Dialectical methods. With this approach, a text can be understood as the result of a dialogue: Concepts, sentences, and lines of thought were established in a discussion of the pros and cons at the expense of alternatives and thus turned out as appropriate or conclusive. Furthermore, dialectics offers methods for summarizing the essential contents of trains of thought.

The fifth approach, hermeneutical methods, will be presented in more detail in the following.

THE PROBLEM OF READING AND THE CONSTRUCTIVE CONTRIBUTION OF HERMENEUTICS

Mathematics is a language of written text. Doing mathematics thus encompasses comprehensive reading of mathematical text. Mathematical language often consists of an intricate, strictly regulated interplay of prosaic and formal language. Importantly, the meaning of mathematical texts is not restricted to their deductive or logical conclusiveness. It rather results from additional, historical aspects of theory and research. As all disciplines are to be reconstructed as practice and under the perspective of their purposes and means, a conclusive understanding of mathematical contents, theorems and even theories presupposes comprehensive knowledge and appraisal of the interests which govern research – as well as those interests which have been supplanted.

As a theory and practice of understanding texts, hermeneutics provides methods to incorporate what has been said by others, especially what has been passed on in writing, into one's own theoretical thinking. These methods relate a text systematically to an individual horizon of understanding. Besides objective understanding, which is constituted by historical reasons and effects, this approach accentuates subjective understanding: gaining access to meaning by starting from prior knowledge in order to subsequently relate "objective and subjective interpretations" to each other. Thus, hermeneutics addresses problems which arise because the language of the arguments in a text is not or only partly one's own language. At the core of hermeneutics is the hermeneutic circle which Hans-Georg Gadamer interprets in his classical work "Truth and method" (Gadamer, 2010) as a process of prior understanding (Vorentwurf), text understanding (Textverstehen), and fusion of horizons (Horizontverschmelzung). In the process of understanding, we inevitably move in a hermeneutic circle because we read texts with our expectations and understand single statements only in the general context and, conversely, this general context only from the single statements.

As a method of text interpretation, the hermeneutic circle allows focused work with mathematical texts, structured reading, slow reading, and self-clarification. Within this circle, students establish connections to their prior knowledge. They can decide how often they take the 3 steps of making prior knowledge explicit, interpreting the text and fusing horizons.[1] The main operation driving interpretation is **translating** text into one's own language. The hermeneutical circle requests the reader to check the suitability of the concepts s/he uses in the translation by making them explicit and showing how a term is a suitable translation or why its meaning cannot (yet) be confirmed with the help of the text. Naming these differences and similarities of text and translation allows putting forward new interpretations and testing hypotheses which might reduce the differences.

EXAMPLE

The exercise presented in the following introduces faculty to hermeneutic interpretation as a mathematical method. The instructors can experiment, train and reflect with material and receive detailed suggestions for their classes. They learn tools for teaching methods of scientific reading and thus systematically supporting the self-regulated, independent reading of their students.

The central idea is to present the instructors with different historical texts which are the starting point of a central mathematical idea. For example, Barrow's proof of a preliminary version of the fundamental theorem of calculus together with the corresponding picture is excellent material for working with instructors in Higher Education (1674/1976, Lectio X, Prop. 11). It is important that the texts are difficult or new enough to put the instructors into a novice's position. In a **first step**, the instructors translate the texts according to their actual, unaided mathematical understanding, similar to the situation of a student. Translations should be demanding enough to make strategic and deliberate work necessary.

In the **second step**, the participants apply text interpretation methods from different philosophical approaches to the same example text. At several independent 'stations', philosophical approaches (as sketched in the introduction) are presented. The task is to apply all the tools to the example and evaluate their benefits relative to the experiences from the first step. As an example, we will describe the hermeneutical station in more detail.

The instructors choose a text and apply the hermeneutic circle to it. They work through the following steps and note their thoughts as inner monologue. For **prior understanding,** they collect all impressions concerning the text including mood. They then express in a preliminary translation hypothesis what the text is about. Thereto, they analyze the ‚prejudices' governing their current understanding. **Text interpretation stage:** With the help of the complete text, the participants test how far their hypothesis summarizes its content and if it deviates from the latter. They validate their impressions with the help of the text. Finally, in the **fusion of horizons**, they summarize their thoughts by enumerating similarities between the text's and the translation's topics. They then start the circle a **second time**, developing a deeper prior understanding. They compare their inner monologue with the translation from the first step which was made without helps. Finally, they discuss the costs and benefits of the hermeneutic circle and how it can be presented in their classes.

The intensive, step-by-step analysis of the text puts the recipient into the role of a reflecting producer who, in a written inner monologue, describes thoughts while translating, making them explicit and, in turn, comprehensible. The writer emphasizes translation and conceptual issues,

[1] The method supports student autonomy. It is especially appropriate as a learning method because it requests the learner to repeatedly evaluate her or his knowledge and skills and reflect the learning process.

compares meanings, evaluates translations and makes the evaluation comprehensible. As the reader may have noted, the example resembles methods from teaching writing, e.g. the exploratory methods developed by John Bean (2001).

Example: “I immediately recognized that it is a mathematical text (prior knowledge). Different from literary texts, formulae are present and persons absent (text interpretation). – Obviously, mathematics is not separable from formulae and very impersonal. Persons don’t play a role. This understanding of mathematics seems to underlie this text – at least at first sight, if I presuppose that we, author and reader, understand the terms “person” and “formula” in the same way (fusion of horizons). Maybe that can be expressed more precisely and I can think of a better and more exact translation (repeated attempt). – I guess that this is related to geometry. The figure in the texts and symbols like AB, CD - these are notions I know from geometry texts (prior knowledge). I am surprised that the figure displays a bent line, not a straight one (text interpretation). That makes me think that …”

The task sketched above already encourages reflection. Nevertheless, a necessary **final step** is meta-reflection. The participants describe and evaluate their experiences with the methods and discuss transfer into their own teaching, preferably in small groups.

OUTLOOK

We suggest that hermeneutic methods – especially the hermeneutic circle – provide a helpful way to read mathematical texts. We elaborated on how, in a manner parallel to student work, instructors can try and test this hermeneutic circle in their own reading of historical texts, focusing on individual prior understanding and its comparison with the text as well as the fusion of horizons. This may foster a deeper understanding of how students decipher texts as well as a direct method of supporting their structured reading of texts. We regard this as an important elementary method of thinking which contributes to comprehensive understanding of mathematical texts.

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