In-depth interviews as a tool in didactics of mathematics

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We aim at characterizing what mathematical experts and novices think about scientific learning and research: about the epistemology of their subject, about relevant skills and about beneficial attitudes, as well as biographical features (dis)advantageous for prolific scientific work. To gain significant qualitative data we adopt techniques for in-depth interviews that include associative and projective methods stemming from systemic and person-centred counselling. One long-term goal is to obtain evidence-based recommendations on how to establish adequate propaedeutics in secondary and tertiary science education in the STEM fields.

Person-centered methods in didactics

Why is it that mathematicians do mathematics? And how do they conceptualise what they are doing? To an outside observer, the behaviour of researchers in, say, pure mathematics might be somewhat puzzling: they struggle with mathematical problems, sometimes for weeks or months, sometimes all by themselves, and mostly with intangible outcomes at best.

A detailed and evidence-based answer to such questions might have consequences for how we teach mathematics, especially on university level, and how we counsel our students with respect to their learning on a cognitive, meta-cognitive as well as emotional level. But there appears to be very little known about these questions in the bold generality in which we have formulated them, apart from occasional anecdotes there seems to be mainly the study of Burton (2004).

There are several equally legitimate points of view on this type of questions, most notably the psychological, concentrating on the individual and what she experiences, and the sociological or ethnographic, focussing on mathematicians as a group with shared narratives and a particular "culture". From both points of view, a quantitative large-scale study[1] should be preceded by a phase of thorough exploration to provide the necessary categories and the necessary theoretical background, and we suggest that person-centered methods are particularly well-suited for this task.

Person-centered methods have a long history in both fields, psychology and ethnography: Person-centered interviews provide a way to treat the interviewee not only as an "informant" who might be asked why he thinks mathematicians do mathematics, but also as a "respondent" who is him- or herself an object of study – observed as she speaks as freely as possible about her own relevant experiences, beliefs and attitudes (Levy and Hollan 1998, Rogers 1957, Langer 2000).

What makes person-centered methods interesting in a didactical setting is the fact that they lend themselves quite easily to being applied in counselling and learning environments (Cornelius-White & Harbaugh 2007). In this presentation, we will highlight person-centered methods as both, empirical and didactical tools.

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The importance of narratives of scientific learning and research

A central part of person-centered interviews is the choice of questions and stimuli designed to help the interviewee to make contact with her inner experience and to freely talk about relevant events, attitudes, beliefs, feelings etc. For us, the most relevant aspects are the internal self-concept of the interviewee as a scientist and how he or she experiences the process of doing mathematics.

In the case of scientists and their individual motivation, there is some evidence that the self-concept is of paramount importance compared to other motivational factors. Indeed, a study of James C. Ryan (2014) on the work motivation of UK-based research scientists (N=405) working in the chemical, biological and biomedical research domains suggests that *internal self-concept motivation* is a key factor for the work motivation of scientists; this source of motivation is one of five compared in the study and "represents an individual's motivation to adhere to their [sic!] internal standards of traits, competencies and values". The other four are (in descending order of their measure of importance) *goal internalisation motivation*, *intrinsic process motivation*, *external self-concept motivation*, and *instrumental motivation*.

These rather general findings do not clarify what the self-concept of a mathematician might actually look like (and it is of course likely that it is not stable over time). The self-concept might be approached by analysing the narratives that scientists tell about their learning and their research. Of course, one should not necessarily take these narratives at face value, but they are of ethnographical interest in their own right and might serve as a starting point for an interview that opens up a space for the interviewee to talk about his or her inner experiences more deeply.

Narratives of scientific learning and research

One type of narrative that can be found in preambles and other parts of school and university curricula is the narrative of mathematics as an instrument to achieve goals that are outside of mathematics, e.g. mathematics as a tool to be used in other sciences or as a means of enhancing argumentative competencies to be employed in other contexts.

We can supplement these narratives with dissenting stories that seem to be highly relevant for the self-concept of prolific scientists.

An archetypical example

Richard Feynman, the world renowned theoretical physicist, gave the following account of his attitude towards his research at Cornell University; there, he had taken up a job in the late 1940ies after having spent several years at Los Alamos, working diligently on the construction of the atomic bomb (Feynman 1985, emph. In original):

Then I had another thought: Physics disgusts me a little bit now, but I used to enjoy doing physics. Why did I enjoy it? I used to play *with it. I used to do whatever I felt like doing – it didn't have to do with whether it was important for the development of nuclear physics, but whether it was interesting and amusing for me to play with. […]*

So I got this new attitude. Now that I am *burned out and I'll never accomplish anything, I've got this nice position at the university teaching classes which I rather enjoy, and just like I*

read the Arabian Nights *for pleasure, I'm going to* play *with physics, whenever I want to, without worrying about any importance whatsoever.*

Feynman went on to not only ponder upon the physics of how flying dinner plates wobble when rotating, but he also did the work on quantum electrodynamics that finally won him the Nobel Prize.

Feynman was, in many ways, not the typical scientist. But this story presents us with an archetype of the kind of rhetoric many scientists might use to describe their perspective on research: The allusion to the polarity of work and play, combined with the expressed belief that this polarity is not a complete description of what is going on in research as play is supposed to be an integral part of how a (pure) scientist might actually work.

Of course, also Feynman would surely admit that play is not the only way to do research – after all, he has been part of the Manhattan project, an endeavour where all means were directed to a single goal, the atomic bomb, and no bit of research was supposed to be an end to itself. But to us, it is an interesting question where eminent scientists position themselves with respect to the relation of work and play in research and how their attitude towards play is related to their very eminence

Rhetorics of play

The narrative of play that we have just met is surely worth to be analysed a little further. In his seminal book "*Homo Ludens*" ("Man the Player"), Johan Huizinga paints the image of play as a driving force behind most if not all human culture (Huizinga 1955). Building on his work, Roger Caillois provides the following, now classic characterisation of play (Caillois 2001):

Play is free, not obligatory; it is separate – circumscribed within limits defined and fixed in advance; it is uncertain, and some latitude for innovation is left to the player's initiative; it is governed by rules; it is accompanied by a special awareness of a second reality; it is unproductive in that it creates no wealth and ends as it begins.

If you replace "play" with "pure scientific research" in this definition then one could argue that you get a reasonably good description of the kind of research Feynman seems to have had in mind (at least if you interpret "unproductive" as "not being intended to be productive").

A more subtle and more diverse image of play can be obtained by considering various *rhetorics of play* as was done by Brian Sutton-Smith (1997) – he lists seven of them. Feynman's narrative evokes what Sutton-Smith calls the *Rhetoric of Self*: it interprets "play in terms of subjective experiences of the player […]; it is an optimal experience, an escape, a release; it is intrinsically motivated […]." This rhetoric has its natural counterpart in a *Rhetoric of Frivolity* that denigrates play as a "waste of time, as idleness, as triviality, and as frivolity", rooted in what might be called a puritan work ethic. The opposition of work and play can itself be analysed as a rhetorical figure and appears to be closely connected to Western culture. So putting *Homo Ludens*, in opposition to *Homo Faber* ("Man the Worker") is itself part of a narrative.

Methods to gather narratives and to facilitate self-clarification

Person-centered interview methods

"Person-centered interviews are a mixture of informant and respondent questions and probes. A probe is an intervention to elicit more information, not necessarily in the form of a question." (Levy and Hollan 1998). The corner stones of person-centered methods are the accepting attitude of the interviewer towards the interviewee and their relationship (Rogers 1957, Langer 2000). "Probes" that might be used in a person-centered interview on the self-concept of mathematicians could comprise:

- A "mathematical fever chart": The interviewee is asked to map his "mathematical biography" on a sheet of paper, as a graph that resembles a fever chart; the time frame could include the time spent at school and university. The particular meaning of "high" and "low" can remain somewhat ambiguous when the task is assigned to the interviewee, the idea being to leave as much space as possible to the interviewee. The interviewee is then asked to name and describe critical points of the graph, giving the interviewer some insight in some emotions and narratives connected to relevant events in the interviewee's biography.
- A collection of narratives is offered to the interviewee. This could take the form of short texts, each on a single card, or even of drawings or other images. The narratives could be taken from Sutton-Smith's list of narratives of play, a list that could actually be read as a list of ways to (consciously or unconsciously) justify acts in general, together with ways to denounce certain acts as immoral or frivolous. The interviewee is now asked to choose cards that he or she considers relevant (for example in light of the biographical information provided in the chart described above). She can now elaborate on how she relates to the narratives on the cards and how her own narratives differ. It is important to include some blank cards to allow the interviewee to substantiate further narratives; more confident interviewees can be asked to draw their own drawings on a card, turning this method into a projective method.
- To increase the authenticity of the situation, the interviewee might first be asked to attack a brain teaser (e.g. a mathematical problem) and to comment on it. It has become apparent that mathematicians quite often react quite strongly and emotionally on mathematical problems that they consider interesting (for further information and a more thorough didactical analysis see (Friedewold & Nicolaisen & Schnieder 2015)). This method could replace the "mathematical fewer chart" in that it provides occasions to talk about narratives of doing mathematics.

These and further person-centered methods can be used in an interview setting – ideally, not only is the audio of the interview transcribed afterwards, but also the relevant paraverbal and nonverbal communication is monitored.

Quite obviously, such an interview can be transformed into a learning environment to enhance the self-awareness of the client by simply replacing the roles of interviewer and interviewee with the roles of teacher/counsellor and student/client.

A pedagogical workshop

The workshop we have planned together with Frauke Link, HTWG Konstanz, is aimed at mathematics lecturers who are ready to examine their mathematical biography. The workshop proceeds on two different levels: First, we try to offer the participants a framework for biographical self-clarification and to ponder on the question: "Why do I do mathematics?" Second, we want to investigate whether the participants' answers contain objectifiable narratives.

The twofold objective of the workshop raises a dilemma: the dilemma between free exploration and structured self-examination. The workshop wants to offer help in a very personal area of self-clarification, in which every participant can explore him- or herself freely and to the depth of his or her choice; the categories to describe this exploration should be found, formulated and explored autonomously.

The subjective diversity that is associated with free self-exploration could of course be avoided by a structured self-examination, in which the categories of self-clarification and self-description are specified in advance and thinking and perception are thus objectively channelled, for instance in the course of a very explicitly structured interview. There does not seem to exist any guideline, any universally approved principles, factors or terminology on which autobiographically oriented attempts to examine oneself could be based and that could be used to separate significant, i.e., transforming experiences in the transition to being a mathematician from "Erfahrungskitsch" ("experience kitsch", Mollenhauer 2008).

The didactical point of our workshop – our attempt to bypass this dilemma – consists in essentially reducing the question "Why do I do mathematics?" to the examination of the following five groups of questions:

- Where did you first encounter mathematics unbraked/unretarded? Which values and which social role was *presented* to you in the process?

- To you, how was mathematics *represented* in school/university? How was mathematics communicated and taught to you there?

- How self-determined were you in developing and contributing your mathematical interests in school/university? When did you become the autonomous subject of your learning of mathematics?

- How much was expected of you as a student? Could it have been more? When was the first time you solved mathematical problems autonomously?

- Do these experiences still have an impact on your mathematical identity today? To what extend?

The point of these five groups of questions is their orientation towards general pedagogical theories (Mollenhauer 2008, Benner 2012). According to these approaches, upbringing and education ("Bildung") are constitutive for how human beings become humans, i.e., for what then realizes itself as (mathematical) personality and identity. Hence, personality as a result of educational processes can be reconstructed as the integration of experiences of an individual with presentation, with representation, as an autonomous subject of its own learning ("Bildsamkeit"), with self-regulated learning ("Selbsttätigkeit") and with identity.

In this respect, they span the elementary topics of the process of biographic self-assurance. If, therefore, Kant (1900) states in the introduction of his lecture on education: “Man can only become man by education. He is merely what education makes of him. It is noticeable that man is only educated by man—that is, by men who have themselves been educated.“, then we conclude from this: human identity forms itself in relation with and delimitation of educational processes faced by the individual. And these processes have an objectifiable basic structure that we try to capture by the five groups of questions that we have given above.

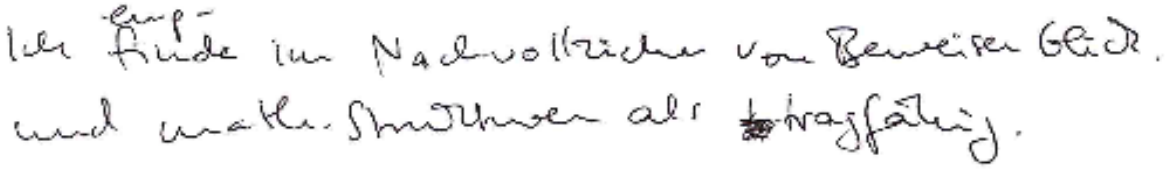
Given the generality but limited number of these questions, we remain this side of a semi-structured interview; we thus avoid the perspective on the individual plurality of biographical formations to be prematurely constricted by predetermined categories. Then again, we should not present the abovementioned questions without comment: we make it clear that the coarsely prestructured questions are meant to partially release the participants from preliminary conceptualizing and analyzing.

First categories found in an explorative study

At a conference on tertiary mathematics education, Frauke Link conducted a workshop, along the above-mentioned lines, with mathematicians (N=15) from several universities in Germany who had varying degrees of experience as lecturers or teachers. The qualitative (written) data that we gathered at the workshop was complemented by three in-depth interviews. From this data, we have extracted a first list of categories that we wish to refine by further qualitative research:

1) *Applications to the real world:* This narrative is characterised by the allusion to real world or scientific applications of mathematics that lie outside mathematics itself. The mathematical language or mathematical results are highly relevant for other fields such as physics, engineering etc. and the relevance of mathematics lies in both, how it increases our capability to understand the world and how it enables us to design products such as computers, cars, etc. Typical examples of statements include “mathematics is the language of reality”.

2) *Reliability of logic:* The characterizing feature of this narrative is that doing mathematics is perceived as something positive because it allows the mathematician to take part in a pure and reliable world. This world is experienced as being supportive, clear, reliable and therefore enjoyable. One is lead to contrast it to the real world which is perhaps perceived as unreliable, unpredictable or obscure. An archetypical example is reproduced below (“I sense happiness when I reconstruct/comprehend proofs and I perceive mathematical structures as capable of bearing.”).



3) *Epistemic curiosity:* This narrative highlights the curiosity of the researcher and her wish to better understand mathematical structures. It embeds into the ancient and venerable narrative of the epistemic curiosity of scientists (“Men pursue science in

order to know, and not for any utilitarian end." – Aristotle, cited after Posnock (1991)), but the objects of curiosity have a very particular, abstract form. "Even today, to recognize and to understand structures is my main motivation when dealing with mathematics."

4) *Meanings and bonds:* One interviewee summarized this narrative as follows: He reported that he was fascinated by mathematical problems and by giving meaning to the mathematical contents he encountered to add: "And that was, many times, a love-hate relationship that […] only develops if you are very closely connected to something. This is, I think, similar as it is in interpersonal relationships."

5) *(Frivolous) play:* This is the narrative mentioned above and exemplified by a quote by Richard Feynman. It was much less prominent than expected when we collected our data in the rather non-directive way described above, but our interviewees could sympathize with it when it was offered to them directly after the actual interview. This may hint to the fact that narratives that may seem socially undesirable are underrepresented in such an open format of inquiry. In a quantitative follow-up to this study, this issue will have to be addressed.

6) *Talent for mathematics:* A narrative that is not typical for mathematics as a science or, in fact, scientific pursuit in general, but can potentially be found in most professions is the narrative of talent. It can take a rather self-determined form ("Because I can do it better than anything else."), but there are variants that allude to a certain lack of autonomy ("It was like sliding into [mathematics] via physics.").

7) *Immediate gratification / flow:* The narrative of mathematical problem solving providing flow experiences (with the solution being immediately gratifying) might seem to be very unspecific at first glance, but there are some special features to it: In school, students might encounter many mathematical tasks that provide clear and immediate feedback (in contrast to tasks in other subjects such as art or history), which could make mathematics gratifying for high-performing students. And you can suspect mathematical problems to allow for flow experiences in a particularly wide range of circumstances (while riding your bicycle, late at night in bed, while painting your walls, etc.).

The audience of the talk kindly suggested two additional narratives that did not show up in our data set so far: *Mathematics as a (competitive) sport* and the *wish to teach mathematics*. The first of the two might be underrepresented as it might have some socially undesirable aspects (at least in a German context). We are curious whether the second of the two will eventually show up as a primary narrative with a sizable number of interviewees, or whether it might have a mostly secondary nature, relying on some other narrative ("… and I would like to pass this on to other people.").

Outlook

Several questions arise naturally from this first part of our study:

- How are the narratives related to the actual experiences of students in school and at university? In what sense might they be "true" and how relevant is this "truth" to the self-image of a person doing mathematics?
- How can we turn this analysis into a valid quantitative tool that can tell us something about the distribution of motivating factors among a population of students?
- If we want our schools and universities to "produce" capable graduates in the STEM-subjects, (how early) should we take the motivational structure of scientists into account?
- Is our list "complete" in the sense that it captures all the relevant narratives?
- If we reframe our research tools as tools of self-clarification, are person-centered methods effective and how much can already be accomplished in a workshop setting?

Next, we are going to focus on the final two questions: A "complete" and accurately defined list of narratives seems to be of value in its own right, and we regard self-clarification as so fundamental for learning a subject that providing effective tools to facilitate it is certainly worthwhile.

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