



Kolloquium des Instituts für Mathematik

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Prime Submodules over Commutative Rings

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This talk presents a systematic study of prime submodules of unital modules over commutative rings, developing module-theoretic analogues of classical prime ideal theory.

A proper submodule N of an R -module M is called *prime* if, for all $r \in R$ and $m \in M$, the condition $rm \in N$ implies either $m \in N$ or $rM \subseteq N$. We first establish foundational examples and show that this notion generalises prime ideals in the ring-as-module case, while pointing out that several ideal-theoretic properties do not transfer verbatim to modules.

We then introduce primary submodules and the associated P -primary terminology, and we characterise prime submodules via the annihilator ideal $(N : M)$: specifically, N is prime if and only if $(N : M)$ is a prime ideal of R and M/N is torsion-free as an $R/(N : M)$ -module. Further characterisations in terms of ideal-submodule products and maximality conditions are obtained.

The talk also examines the radical of a submodule, denoted $\text{rad } N$, defined as the intersection of all prime submodules containing N . Key properties are established, including a sum formula for finitely generated modules:

$$\text{rad } N + \text{rad } L = M \iff N + L = M.$$

Chain conditions are examined next: for a totally ordered family of prime submodules, their intersection is again prime, and their union is prime when M is finitely generated. These results culminate in a Cohen-type theorem for modules: a finitely generated R -module M is Noetherian if and only if every prime submodule of M is finitely generated. A counterexample based on the $\mathbb{Z}$ -module $\mathbb{Q}$ demonstrates the necessity of finite generation.

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16 Uhr c.t.

gez. Prof. Dr. J. Prestin