Neighborhood structures and products of undirected graphs

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Abstract

Let $G = (V, E)$ be a simple undirected graph. The \textit{neighborhood hypergraph} $\mathcal{N}(G) = (V, \mathcal{E}^N)$ of $G$ has the edge set $\mathcal{E}^N = \{ e \subseteq V \mid |e| \geq 1 \land \exists x \in V : e = N_G(x) \}$. In a certain sense, this is a generalization of the well-known notion of the \textit{neighborhood graph} $N(G) = (V, E_N)$. For several products $G_1 \circ G_2$ of simple undirected graphs $G_1$ and $G_2$, we investigate the question how $N(G_1 \circ G_2) / N(G_1 \circ G_2)$ can be constructed from $G_1$, $G_2$, $N(G_1)$, $N(G_2)$, $N(G_1)$, $N(G_2)$ and vice versa.

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