Iterated neighborhood graphs

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Abstract

Let $G = (V, E)$ be a simple undirected graph. $N(G) = (V, E_N)$ is the neighborhood graph of $G$, if and only if

$$E_N = \{\{a, b\} \mid a \neq b \land \exists x \in V : \{x, a\} \in E \land \{x, b\} \in E\}.$$

It is well-known that the neighborhood graph $N(G)$ is connected if and only if the graph $G$ is connected and non-bipartite. We present some results concerning the $k$-iterated neighborhood graph $N^k(G) := N(N(...N(G)))$ of $G$. In particular we investigate conditions for $G$ and $k$ such that $N^k(G)$ becomes a complete graph.

Keywords: neighborhood graph, 2-step graph, neighborhood completeness number.

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