Stability of translates of scaling vectors

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Usually, the construction of multiwavelets is based on a multiresolution analysis (MRA) with higher multiplicity. In order to generate the MRA, a refinable function vector $\boldsymbol{\phi} := (\phi_0, \ldots, \phi_{r-1})^T$ is needed, such that $\{\phi_\nu(\cdot - l) : l \in \mathbb{Z}, \nu = 0, \ldots, r-1\}$ forms a Riesz–basis of its span. Moreover, the components of $\boldsymbol{\phi}$ are often desired to be compactly supported, regular and symmetric such that multiwavelets $\boldsymbol{\psi}$ with similar properties can be derived.
The function vector $\boldsymbol{\phi}$ can be considered as a solution of a functional equation

$$\boldsymbol{\phi}(x) = \sum_{l \in \mathbb{Z}} \boldsymbol{P}_l \, \boldsymbol{\phi}(2x - l), \tag{1}$$

where $\boldsymbol{P}_l$ are $(r \times r)$ coefficient matrices. Hence, the question occurs, how a desired solution vector $\boldsymbol{\phi}$ of (1) can be found, just by appropriate choice of the symbol

$$\boldsymbol{P}(\omega) := \sum_{l \in \mathbb{Z}} \boldsymbol{P}_l \, e^{-i\omega l}.$$

The talk summarizes necessary and sufficient conditions for $\boldsymbol{P}(\omega)$ yielding a compactly supported solution vector $\boldsymbol{\phi}$ which satisfies the Riesz–basis property.