Mecklenburg Workshop

Approximation Methods
and
Fast Algorithms

Hasenwinkel
September 10–14, 2018

Organizers
Gerlind Plonka-Hoch (Georg-August-Universität Göttingen)
Daniel Potts (Technische Universität Chemnitz)
Jürgen Prestin (Universität zu Lübeck)
Gabriele Steidl (Technische Universität Kaiserslautern)
Dedicated to the 75th birthday of
Prof. Dr. Manfred Tasche
# Program Overview

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All **talks** will take place at *Remise*

**Breakfast** and **Lunch** will be served at the dining room at *Schloss*

**Dinner** will be served at *Marstall* or at *Schloss*
Program
Monday

16.00 - 16.30  Opening
Chair: Stefan Kunis

16.30 - 17.00  Lutz Kämmerer
Fast Fourier Transform methods for sparse multivariate
trigonometric polynomials and approximation properties

17.00 - 17.30  Robert Nasdala
Transformed rank-1 lattices for high-dimensional approximation

17.30 - 18.00  Matthias Beckmann
Saturation rates for filtered back projection

18.00 - 18.30  Kevin Schober
Multivariate directional wavelet systems on the torus
### Tuesday

**Chair:** Armin Iske

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<td>Robert J. Kunsch</td>
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<td>Dirk Langemann</td>
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<td>Ronny Bergmann</td>
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<td>Michael Skrzipek</td>
<td>Zeros of polynomials used in frequency analysis</td>
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<td>Andrii L. Shidlich</td>
<td>Direct and inverse approximation theorems of functions in the Orlicz type spaces</td>
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<td>Serhii Stasyuk</td>
<td>Sparse trigonometric approximation of periodic functions with mixed smoothness</td>
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Wednesday

Chair: Frank Filbir

09.00 - 10.00 **Lothar Reichel**
*Generalized Krylov subspace methods for $\ell_p$-$\ell_q$ minimization with application to image restoration*

10.00 - 10.30 Sara Krause-Solberg
*A tractable approach for 1-bit compressed sensing on manifolds*

10.30 - 11.00 **Coffee Break**

Chair: Ute Schreiber

11.00 - 11.30 Florian Boßmann
*Shifted rank-1 approximation and its application in Geophysics*

11.30 - 12.00 Stefan Alexander Kahler
*On nonnegative linearization of orthogonal polynomials*

12.00 - 12.30 Michael Quellmalz
*The cone-beam transform and spherical convolution operators*

12.30 - 14.30 **Lunch**

Chair: Michael Skrzipek

14.30 - 15.00 Marzieh Hasannasab
*Frames and operator representations of frames*

15.00 - 15.30 Ralf Hielscher
*Isomorphic embeddings of quotients of the rotation group*

15.30 - 16.00 **Coffee break**

16.00 **Walking-tour**

19.00 **Conference Dinner**
Thursday morning

Chair: Manfred Tasche

09.00 - 10.00 Frank Filbir
Phase retrieval from short-time Fourier measurements

10.00 - 10.30 Robert Beinert
Phase retrieval of non-uniform splines by Prony’s method

10.30 - 11.00 Coffee Break

Chair: Dirk Langemann

11.00 - 11.30 Thomas Peter
A randomized multivariate matrix pencil method

11.30 - 12.00 Hanna Veselovska
Bivariate Prony-type polynomials

12.00 - 12.30 Dominik Nagel
Condition numbers of Vandermonde matrices with nearly-colliding nodes

12.30 - 14.30 Lunch
Thursday afternoon

Chair: Peter Junghanns

14.30 - 15.30  **Yurii Kolomoitsev**
*Approximation by generalized Kantorovich-Kotelnikov sampling operators*

15.30 - 16.00  Gisela Pöplau
*More trouble with tribbles – Investigation on the ion motion towards clearing electrodes in a particle accelerator*

16.00 - 16.30  *Coffee break*

Chair: Karla Rost

16.30-17.00  Sina Bittens
*Real sparse fast DCT for vectors with short support*

17.00-17.30  Melanie Kircheis
*A frame-theoretical approach to the inversion of the NFFT*

17.30-18.00  Franziska Nestler
*Parameter tuning for the nonuniform FFT and applications*

18.00-18.30  Manuel Gräf
*Nonequispaced fast Fourier transforms on the Grassmannian manifold $G_{2,4}$*
Friday

Chair: Lothar Reichel

09.00 - 10.00  **Armin Iske**  
Model-based design of kernel approximation methods for data analysis

10.00 - 10.30  Peter Junghanns  
An integral equation to minimize airplane drag

10.30 - 11.00  **Coffee break**

Chair: Michael Eiermann

11.00 - 11.30  Markus Weimar  
On the regularity of solutions to quasi-linear PDEs

11.30 - 12.00  Martin Ehler  
Discrepancy kernels on the special orthogonal group and the Grassmannian
Plenary Talks

Tuesday, 09.00 - 10.00
Frames and dynamical sampling
OLE CHRISTENSEN
Technical University of Denmark

The talk will give a short survey to frame theory in Hilbert spaces, followed by a more detailed discussion of the recent research topic "dynamical sampling." Formulated in purely mathematical terms, the key question is when and how a frame can be represented via iterations of a certain bounded operator, acting on a fixed vector in the underlying Hilbert space. The talk presents joint work with Marzieh Hasannasab.

Thursday, 09.00 - 10.00
Phase Retrieval from Short-Time Fourier Measurements
FRANK FILBIR
Helmholtz Center Munich & Technische Universität München

Phase retrieval in its general form refers to the problem of recovering a signal \( f \) from the magnitude of its frame coefficients \( |\langle f, \varphi_m \rangle| \). In the classical setting the analysing frame \( \varphi_m \) is given as the Fourier basis. The PR problem is of great interest in various fields of applied science like crystallography, diffraction imaging and many more. Recently a new imaging method, now known as ptychographical imaging, was developed. This diffraction based imaging technique works with time-frequency frames (or, more precisely space-frequency frames). That means we are given measurements of the form \( |\langle f, \varphi_{n,m} \rangle|^2 \), where \( (\varphi_{n,m}) \) is a time-frequency frame. The aim is to reconstruct \( f \).

In this talk we will give some background on the physics and a short review on existing mathematical methods before we present some current work on the problem. The talk is based on collaboration with Mark Iwen (Michigan State University), Rayan Saab (UC San Diego), and the group of Christian Schroer (DESY, Hamburg).
Friday, 09.00 - 10.00
Model-based design of kernel approximation methods for data analysis

Armin Iske
Universität Hamburg

We show how to design kernel-based approximation schemes in situations where standard kernel methods are not suitable or even doomed to fail. To this end, we first discuss the characterization and construction of non-standard kernels for data analysis on the basis of specific model assumptions. This leads us to a larger class of flexible kernel-based approximation methods, relying on weighted kernels that are symmetric but not radially symmetric. Further in our discussion, we then turn to the design of kernel-based regularization methods, as they are relevant in applications of machine learning. We address numerical aspects and, moreover, we provide numerical examples for further illustration.

Thursday, 14.30 - 15.30
Approximation by generalized Kantorovich-Kotelnikov sampling operators

Yurii Kolomoitsev
Universität zu Lübeck

We study approximation properties of the generalized Kantorovich-Kotelnikov sampling operator given by

\[ Q_j(f, \varphi, \tilde{\varphi}; x) = \sum_{k \in \mathbb{Z}^d} \left( m^j \int_{\mathbb{R}^d} f(u)\tilde{\varphi}(M^j u + k)du \right) \varphi(M^j x + k), \]

where \( M \) is a dilation matrix, \( m = |\det M| \), and \( \tilde{\varphi} \) and \( \varphi \) are appropriate functions. In particular, we consider the case in which the function \( \varphi \) belongs to a certain subspace of band-limited functions including non-integrable ones and \( \tilde{\varphi} \) belongs to a subspace of \( L_q \) containing, in particular, compactly supported functions. Under certain smoothness and compatibility conditions on \( \tilde{\varphi} \) and \( \varphi \), we obtain several types of estimates for the error \( \| f - Q_j(f, \varphi, \tilde{\varphi}; \cdot) \|_{L^p(\mathbb{R}^d)} \) in terms of the classical and anisotropic moduli of smoothness, the error of the best approximation, and in terms of decay of the Fourier transform of the function \( f \). Different examples of the Kantorovich-Kotelnikov operators generated by the sinc-function and its linear combinations will be also presented.

This is joint work with Maria Skopina and Aleksandr Krivoshein (St. Petersburg State University).
Wednesday, 09.00 - 10.00

**Generalized Krylov subspace methods for $\ell_p$-$\ell_q$ minimization with application to image restoration**

**Lothar Reichel**  
Department of Mathematical Sciences, Kent State University, Kent, OH 44242, USA

This talk presents new efficient approaches for the solution of $\ell_p$-$\ell_q$ minimization problems based on the application of successive orthogonal projections onto generalized Krylov subspaces of increasing dimension. The subspaces are generated according to the iteratively reweighted least-squares strategy for the approximation of $\ell_p$- and $\ell_q$-norms or quasi-norms by using weighted $\ell_2$-norms. Computed image restoration examples illustrate the performance of the methods discussed. The talk presents joint work with A. Buccini, G.-X. Huang, A. Lanza, S. Morigi, and F. Sgallari.

Tuesday, 14.30 - 15.30

**Restricted inversion of split-Bezoutians**

**Karla Rost**  
Technische Universität Chemnitz

The main aim of this talk is to present inverses of split-Bezoutians considered as linear operators restricted to subspaces of symmetric or skewsymmetric vectors. Such results are important, e.g., for the inversion of nonsingular, centrosymmetric or centroskewsymmetric Toeplitz-plus-Hankel Bezoutians $B$ of order $n$. To realize this inversion we present algorithms with $O(n^2)$ computational complexity, which involves an explicit representation of $B^{-1}$ as a sum of a Toeplitz and a Hankel matrix.

Based on different ideas such inversion formulas have already been proved in previous joint papers with Torsten Ehrhardt.

But now we focus on the occurring splitting parts since they are of interest also in a more general context. The main key is the solution of the converse problem: the inversion of Toeplitz-plus-Hankel matrices, where we use classical results of joint work with Georg Heinig. An advantage of our approach here is that all appearing special cases can be dealt with in the same, relatively straightforward way without any additional assumptions.

The presented results were obtained together with Torsten Ehrhardt.
Contributed Talks

Monday, 17.30 - 18.00
Saturation Rates for Filtered Back Projection
MATTHIAS BECKMANN
Department of Mathematics, University of Hamburg

The method of filtered back projection is a well-known and commonly used reconstruction technique in computerized tomography, which allows us to recover bivariate functions from given Radon samples. The reconstruction is based on the classical filtered back projection (FBP) formula, which gives an analytical inversion of the Radon transform. The FBP formula, however, is numerically unstable and suitable low-pass filters of finite bandwidth and with a compactly supported window function are employed to make the reconstruction by FBP less sensitive to noise. This leads to an approximate reconstruction formula.

In this talk we analyse the inherent FBP reconstruction error which is incurred by the application of a low-pass filter. To this end, we present error estimates in Sobolev spaces of fractional order, where the obtained error bounds depend on the bandwidth of the utilized filter, on the flatness of the filter’s window function at the origin, on the smoothness of the target function, and on the order of the considered Sobolev norm. Furthermore, we prove convergence for the approximate FBP reconstruction in the treated Sobolev norms along with asymptotic rates of convergence as the filter’s bandwidth goes to infinity, where we observe saturation at fractional order depending on smoothness properties of the filter’s window function. The theoretical results are supported by numerical experiments.

This talk is based on joint work with Armin Iske.

Thursday, 10.00 - 10.30
Phase retrieval of non-uniform splines by Prony’s method
ROBERT BEINERT
Karl-Franzens-Universität Graz

The phase retrieval problem consists in recovering a complex-valued signal from the modulus of its Fourier transform. In other words, the phase of the signal in the frequency domain is lost. Recovery problems of this kind have many applications in physics and engineering as for example in electron microscopy, crystallography, astronomy, and communications. The long history of phase retrieval include countless approaches to find an analytic or a numerical solution, which is generally challenging due to the well-known ambiguousness of the problem.

In this talk, we consider the one-dimensional continuous-time phase retrieval problem, where we wish to recover a complex-valued signal \( f : \mathbb{R} \rightarrow \mathbb{C} \) from its Fourier intensity \( |\mathcal{F}[f]| \). In addition, we assume that the true signal \( f \) has a sparse representation of the form

\[
f(t) = \sum_{j=1}^{N} c_j^{(0)} \delta(t - T_j) \quad \text{or} \quad f(t) = \sum_{j=1}^{N} c_j^{(m)} B_{j,m}(t),
\]

where \( \delta \) denotes the Delta distribution, and \( B_{j,m} \) the B-spline of order \( m \) determined by the knots \( T_j < T_{j+1} < \cdots < T_{j+m} \). The main question is now: can we always recover the unknown complex coefficients \( c_j^{(m)} \) and the real knots \( T_j \) from the given Fourier intensity?

Using a constructive proof, we show that almost all sparse signals \( f \) consisting of \( N \) spikes at arbitrary locations can be uniquely recovered up to trivial ambiguities—up to rotations,
time shifts, and conjugated reflections. An analogous result holds for spline functions of order \(m\). The proof itself consists of two main steps. Exploiting that the autocorrelation function \(\hat{a}\) of the sparse signal \(f\) is here always an exponential sum, we firstly apply Prony’s method to recover the unknown parameters (coefficients and frequencies) of \(\hat{a}\). In a second step, we use this information, which already comprise all occurring knot differences \(T_j - T_k\), to derive the unknown parameters of the signal \(f\).

On the basis of the proof, we moreover present an algorithm to recover \(f\) from \(O(N^2)\) intensity measurements \(|F[f](\omega)|\) of its Fourier transform. Finally, we illustrate the proposed method at different numerical examples.

Tuesday, 11.30 - 12.00

A variational model for data fitting on manifolds by minimizing the acceleration of a Bézier curve

RONNY BERGMANN
Technische Universität Chemnitz

Fitting a smooth curve to data points \(d_0, \ldots, d_n\) lying on a Riemannian manifold \(\mathcal{M}\) and associated with real-valued parameters \(t_0, \ldots, t_n\) is a common problem in applications like wind field approximation, rigid body motion interpolation, or sphere-valued data analysis. The resulting curve should strike a balance between data proximity and a smoothing regularization constraint.

In this talk we present a variational model to fit a composite Bézier curve to the set of data points \(d_0, \ldots, d_n\) on a Riemannian manifold \(\mathcal{M}\). The resulting curve is obtained in such a way that its mean squared acceleration is minimal in addition to remaining close the data points. We approximate the acceleration by discretizing the squared second order derivative along the curve. We derive a closed-form, numerically stable and efficient algorithm to compute the gradient of a Bézier curve on manifolds with respect to its control points. This gradient can be expressed as a concatenation of so called adjoint Jacobi fields. Several examples illustrate the capabilities of this approach both for interpolation and approximation.

This is joint work with P.-Y. Gousenbourger.

Thursday, 16.30 - 17.00

Real Sparse Fast DCT for Vectors with Short Support

SINA BITTENS
University of Göttingen

There are many well-known fast algorithms for the discrete Fourier transform (DFT) of sparse input functions or vectors, both deterministic and randomized ones. Under the assumption that the \(N\)-length input vector has a short support of length \(m\), the fastest of these algorithms achieve runtimes of \(O(m \log N)\).

For the closely related discrete cosine transform of type II (DCT-II), given by

\[
\hat{x}^{\text{II}} := \sqrt{\frac{2}{N}} \left( \varepsilon_N(j) \cos \left( \frac{j(2k+1)\pi}{2N} \right) \right)_{j,k=0}^{N-1} x, \quad x \in \mathbb{R}^N,
\]

where \(\varepsilon_N(0) = 1/\sqrt{N}\) and \(\varepsilon_N(j) = 1\) for \(j \neq 0\), there also exists a fast algorithm with a runtime of \(O(m \log m \log N)\) if \(x\) has a short support of length \(m\) (see Bittens and Plonka, Sparse Fast DCT for Vectors with One-block Support, http://arxiv.org/abs/1803.05207,
2018). However, this algorithm employs complex arithmetic utilizing the close relation between the DCT-II and the DFT, even though the DCT itself is a completely real transform.

In this talk we present a new fast and deterministic IDCT-II algorithm that reconstructs the input vector $x \in \mathbb{R}^N$, $N = 2^J$, with short support of length $m$ from $x^\mathbb{D}$ using only real arithmetic if an upper bound $M$ on $m$ is known. The resulting algorithm has a runtime of $O\left( M \log M + m \log_2 \frac{N}{M} \right)$, requires $O\left( M + m \log_2 \frac{N}{M} \right)$, and does not employ inverse FFT algorithms to recover $x$.

**Joint work with:** Gerlind Plonka.

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**Wednesday, 11.00 - 11.30**

**Shifted rank-1 approximation and its application in Geophysics**

**FLORIAN BOSSMANN**

*Harbin Institute of Technology*

Seismic exploration seeks to find subsurface deposits of e.g., oil, gas or minerals using seismic waves. Processing and analyzing the obtained data is an important but difficult task. Many developed methods rely on the assumption that data is of low complexity in some domain. In former works many different domains such as Wavelets, Shearlets or adaptive dictionaries have been considered. However, often the methods are designed to achieve a sparse approximation where the physical background only takes a minor role.

We present a method which is designed to return a sparse approximation of given data where each element represents a unique seismic event. Thus, directly giving information about its origin. Let $A \in \mathbb{R}^{m \times n}$ be the obtained data matrix where each column represents a time signal measured at a fixed spatial position. We approximate the data as

$$A \approx \sum_{k=1}^{L} S_{\lambda_k}(u_k v_k^*)$$

where $u_k \in \mathbb{R}^m$, $v_k \in \mathbb{R}^n$ form a rank-1 matrix and $S_{\lambda_k}$ is a linear operator that shifts each column of the matrix by a value defined by $\lambda_k \in \mathbb{Z}^n$. The number of events $L$ is small. In this setting, $u_k$ can be interpreted as seismic wave, $v_k$ as its amplitude at different positions and $\lambda_k$ as its arrival time. The model is related to some shift invariant dictionary learning techniques using an additional uniqueness assumption. This is joint work with Jianwei Ma.

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**Friday, 11.30 - 12.00**

**Discrepancy kernels on the special orthogonal group and the Grassmannian**

**MARTIN EHLER**

*University of Vienna, Department of Mathematics*

Given a Borel probability measure $\mu$ on $\mathbb{R}^d$ with compact support $S := \text{supp}(\mu)$, a fundamental sampling problem is to allocate a suitable $n$-point set $\{x_1, \ldots, x_n\} \subset S$ such that the normalized counting measure

$$\nu_n := \frac{1}{n} \sum_{j=1}^{n} \delta_{x_j}$$

approximates $\mu$. Positive-definite discrepancy kernels $K : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ induce a quantification of the distance between $\nu_n$ and $\mu$. The measure $\nu_n$ is then constructed by numerically minimizing a suitable optimization problem $D(\mu, K, n)$ among all $n$-point sets.
\{x_1, \ldots, x_n\} \subset S$ for fixed $n \in \mathbb{N}$. Access to the Fourier expansion of $K|_{S \times S}$ enables the use of the nonequispaced fast Fourier transform [4] within the numerical routines, thereby offering faster and more efficient minimization of $\mathcal{D}(\mu, K, n)$. Hence, we must

i) compute the Fourier coefficients of $K|_{S \times S}$.

Minimizing $\mathcal{D}(\mu, K, n)$ relates to the construction of quadrature points for integrating functions in the reproducing kernel Hilbert space $\mathcal{H}(S)$ generated by $K|_{S \times S}$. Therefore, we aim to identify $\mathcal{H}(S)$ with a classical function space. Sobolev smoothness is generally quantified by Fourier decay properties, so that we aim to

ii) determine the asymptotics of $K|_{S \times S}$’s Fourier coefficients.

For $S = \mathbb{S}^{d-1}$ and a particular choice of $K$, the items i) and ii) are addressed in [1, 3]. In this talk, we shall discuss analogous results for $S$ being the special orthogonal group and the Grassmannian manifold,

$$\text{SO}(d) := \{x \in \mathbb{R}^{d \times d} : \det(x) = 1, x^{-1} = x^\top\},$$
$$G_{k,d} := \{x \in \mathbb{R}^{d \times d} : x^\top = x, x^2 = x, \text{trace}(x) = k\}.$$

While SO(3) is partially covered by [2], we shall focus on $G_{2,4}$. Note that the nonequispaced fast Fourier transform is available for SO(3), see [5], and, as of late, also for $G_{2,4}$.

(This is joint work with Josef Dick, Manuel Gräf, and Christian Krattenthaler)

REFERENCES


Tuesday, 15.30 - 16.00

**Computing the derivative of a matrix function, with an application to parameter identification problems for the heat equation**

**MICHAEL EIERMANN**

*Institut für Numerische Mathematik und Optimierung, TU Bergakademie Freiberg*

Let $A = A(c)$ be an $n \times n$-matrix depending on a parameter $c$ and let $b$ be an $n$-dimensional vector. New algorithms are developed for computing the matrix-vector products $v \mapsto Jv$ and $w \mapsto J^T w$, where $J$ denotes the derivative of $u(t, c) = \exp(tA(c))b$ with respect to $c$. We assume that $A$ is large and sparse, i.e., none of the large, but dense matrices,
exp(tA) and J, can be computed explicitly or stored. Our algorithms are based on the approximation of the exponential function by the partial sums of its Taylor or Chebyshev series and require only matrix-vector multiplications by A. To avoid numerical cancellation, a scaling strategy is included. Alternatively, we consider methods which rely on the rational best approximation of the exponential function on (−∞, 0]. We analyze the truncation and backward errors of these algorithms and describe their behavior in finite precision arithmetic.

Thursday, 18.00 - 18.30
Nonequispaced fast Fourier transforms on the Grassmannian manifold $G_{2,4}$

**MANUEL GRÄF**

Acoustics Research Institute - Austrian Academy of Sciences

During the last decades, fast Fourier transforms for nonequispaced sampling points have been developed for the torus $\mathbb{T}^d$, the sphere $S^2$ and the rotation group $SO(3)$. These algorithms have been proven successful in solving large scale problems in medical imaging, material physics and molecular dynamics, and many others.

As a matrix manifold the Grassmannian manifold

$$G_{k,d} := \{ P \in \mathbb{R}^{d \times d} \mid P^T = P, \quad P^2 = P, \quad \text{trace}(P) = k \}$$

has become more attractive in approximation theory, so that efficient computational algorithms need to be implemented. In particular, we are interested in fast evaluation and reconstruction methods of polynomials. The space of polynomials of degree at most $t$ on $G_{k,d}$ is denote by $\text{Pol}_t(G_{k,d})$ and consists of all multivariate polynomials in the matrix entries of total degree at most $t$.

In this talk we consider the Grassmannian $G_{2,4}$, which is the first case that cannot be reduced to one of the cases $\mathbb{T}^d$, $S^2$, $SO(3)$. We shall present a pleasant parameterization $P : S^2 \times S^2 \rightarrow G_{2,4}$ which reflects the fact that $G_{2,4} \cong S^2 \times S^2 / \{ \pm 1 \}$. Moreover, it enables us for the construction of an explicit basis $Y_n$, $n = 1, \ldots, N$, of $\text{Pol}_t(G_{2,4})$, based on tensor spherical harmonics. For given sampling points $P_i \in G_{2,4}$, $i = 1, \ldots, M$, these particular choices allow for the fast evaluation of the following sums

$$f_i = \sum_{n=1}^{N} \hat{f}_n Y_n(P_i), \quad i = 1, \ldots, M, \quad \hat{f}_n = \sum_{i=1}^{M} f(P_i) Y_n(P_i), \quad n = 1, \ldots, N.$$

The naive computation of such sums takes $O(NtM)$ operations. We provide approximate algorithms with complexity $O(Nt \log^3(Nt) + M |\log \epsilon|^4)$, where $\epsilon$ is an approximation error. The corresponding algorithms are called the nonequispaced fast Fourier transform on $G_{2,4}$ and the nonequispaced fast adjoint Fourier transform on $G_{2,4}$, for the first and latter sum, respectively. These fast Fourier transforms utilize the NFFT library [1] and are implemented in a Python library, which will be publicly available.

**REFERENCES**

Wednesday, 14.30 - 15.00  
**Frames and operator representations of frames**  
MARZIEH HASANNASAB  

*Technical University of Denmark*

One of the central questions in dynamical sampling is to identify when and how a given frame can be represented via iterated actions of a bounded operator on a single element in the underlying Hilbert space. The talk will provide various characterizations of the frames for which this can be done. The talk presents joint work with Ole Christensen.

Wednesday, 15.00 - 15.30  
**Isomorphic Embeddings of Quotients of the Rotation Group**  
RALF HIELSCHER AND LAURA LIPPERT  
*TU Chemnitz, Germany*

Quotients $SO(3)/S$ of the rotation group $SO(3)$ with respect to finite symmetry groups $S \subset SO(3)$ play an important role in crystallography, robotics and molecular science. Although the analysis of manifold valued data has seen big progressions during the last years there are still many methods which are only applicable to data in Euclidean vector spaces. For this reason we are interested in embeddings of the quotients $SO(3)/S$ into some vector space $\mathbb{R}^d$. According to the Whitney embedding theorem such embeddings exist for $d \geq 6$. Explicit embeddings for $SO(3)/S$ have been reported in [1]. In our talk we generalize the construction of these embeddings which allows us to make them isomorphic.


Friday, 10.00 - 10.30  
**An integral equation to minimize airplane drag**  
PETER JUNGHANNS  
*Technische Universität Chemnitz*

We consider an airplane wing, not necessarily symmetric, or a system of such wings, for which the optimal circulation distribution has to be determined. This latter is the solution of a constraint minimization problem, whose Euler-Lagrange equation is a (system of) Cauchy singular integral equation(s). We obtain existence and uniqueness results in suitable weighted Sobolev type spaces. Then, we propose a collocation-quadrature method to solve the problem numerically and prove stability, convergence and convergence rates. Some numerical examples, which confirm the previous error estimates, are also presented.

This talk is based on joint work (cf. [1]) with G. Monegato, Dipartimento di Scienze Mathematiche, Politecnico di Torino, Turin, Italy, and L. Demasi, Department of Aerospace Engineering, San Diego State University, San Diego, CA, USA.

On nonnegative linearization of orthogonal polynomials

STEFAN ALEXANDER KAHLER
RWTH Aachen University

Besides their central role in the theory of special functions, orthogonal polynomials have several important applications in related fields such as approximation theory or numerical mathematics, in particular when the polynomials are accompanied by an appropriate Fourier or harmonic analysis. An elegant way to relate orthogonal polynomials to Gelfand theory and further concepts of harmonic analysis has been provided by the concept of a polynomial hypergroup, introduced by R. Lasser in the 1980s. This concept allows to consider a corresponding Fourier and Plancherel transformation in a very unified way but also offers a great variety because the individual behavior strongly depends on the underlying polynomials.

The sequences \((P_n(x))_{n \in \mathbb{N}_0}\) of orthogonal polynomials which underlie polynomial hypergroups satisfy the crucial nonnegative linearization property that, under a suitable normalization, the product of any two polynomials \(P_m(x), P_n(x)\) is a convex combination of the polynomials \(P_{|m-n|}(x), \ldots, P_{m+n}(x)\). Given a concrete sequence, verifying or disproving this property may be very delicate. In the 1970s, G. Gasper solved the problem for the Jacobi polynomials \((R_n^{(\alpha,\beta)}(x))_{n \in \mathbb{N}_0}\) by specifying the parameters \(\alpha, \beta > -1\) for which the linearization property holds. In the “open problems section” of (2), R. Szwarc asked to solve the analogous problem concerning the generalized Chebyshev polynomials \((T_n^{(\alpha,\beta)}(x))_{n \in \mathbb{N}_0}\), which are the quadratic transformations of the Jacobi polynomials and orthogonal w.r.t. the measure \(d\mu(x) = (1 - x^2)\alpha|x|^{2\beta+1}\). Another large class is given by the associated symmetric Pollaczek polynomials, which is a two-parameter generalization of the class of ultraspherical polynomials and whose orthogonal polynomial sequences are—in the monic normalization—given by recurrence relations of the form \(p_0(x) = 1, p_1(x) = x, xp_n(x) = p_{n+1}(x) + \frac{(n + \nu)(n + \nu + 2\alpha)}{(2n + 2\nu + 2\alpha + 2\lambda + 1)(2n + 2\nu + 2\alpha + 2\lambda + 1)} p_{n-1}(x)\) with \(\alpha > -\frac{1}{2}\) and \(\lambda, \nu \geq 0\). These polynomials are of particular interest with regard to asymptotic behavior; a conjecture concerning the parameter region for which the linearization property holds was made by Lasser in (1). In this talk, we present a full solution to the problem posted by Szwarc and establish a stronger result than Lasser’s conjecture by showing that all parameters \(\alpha > -\frac{1}{2}, \lambda, \nu \geq 0\) have the desired property.


Monday, 16.30 - 17.00

**Fast Fourier Transform Methods for Sparse Multivariate Trigonometric Polynomials and Approximation Properties**

LUTZ KÄMMERER  
*Universität Osnabrück*

We discuss three different structures of spatial discretizations that allow for fast Fourier transform algorithms in high dimensional settings. Starting with sparse grids and single rank-1 lattices, we highlight advantages and disadvantages of these two kinds of spatial discretization.

A recently published third idea uses sampling values along multiple rank-1 lattices in order to reconstruct multivariate trigonometric polynomials. Efficient methods for the construction of these spatial discretizations as well as fast Fourier transform algorithms are already available.

Various numerical tests promise that the new sampling method seems to avoid the disadvantages and combine the advantages of sparse grid sampling and rank-1 lattice sampling. This motivates further investigations on the corresponding method, in particular on the approximation properties of the emerging sampling operators applied on functions of generalized mixed smoothness.

We compare the three different sampling methods with respect to their approximation properties in detail. For instance, when using multiple rank-1 lattices and measuring the sampling error in the $L^2$-norm, we show sampling error estimates where the exponent of the main part reaches those of the optimal sampling rate, which is obtained by the sparse grid sampling method, except for an offset of $1/2 + \varepsilon$, i.e., the exponent is almost a factor of two better up to the mentioned offset compared to single rank-1 lattice sampling.

This is joint work with Toni Volkmer.

Thursday, 17.00 - 17.30

**A frame-theoretical approach to the inversion of the NFFT**

MELANIE KIRCHEIS  
*Technische Universität Chemnitz*

The NFFT, short hand for nonequispaced fast Fourier transform, is a fast algorithm to evaluate a trigonometric polynomial

$$ f(x) = \sum_{k=-M/2}^{M/2-1} \hat{f}_k e^{2\pi i k x} $$

at nonequispaced points $x_j \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, $j = 1, \ldots, N$. In case we are given equispaced points and $M = N$, this evaluation can be realized by means of the FFT, a fast algorithm that is invertible. Hence, we are interested in an inversion also for nonequispaced data, i.e., the Fourier coefficients $\hat{f}_k$ shall be computed for given function values $f(x_j)$.

For this purpose, we use the matrix representation of the NFFT to derive a direct algorithm. Thereby, we are able to compute an inverse NFFT by dint of a modified adjoint NFFT. Finally, we show that this approach can also be explained by means of frame approximation.
Wednesday, 10.00 - 10.30
A tractable approach for 1-bit compressed sensing on manifolds

SARA KRAUSE-SOLBERG
TU München

I will discuss the problem of recovering a signal from one-bit compressed sensing measurements under a manifold model; that is, assuming that the signal lies on or near a manifold of low intrinsic dimension. I present a convex recovery method based on the Geometric Multi-Resolution Analysis with proven recovery guarantees with a near-optimal scaling in the intrinsic manifold dimension. The method is the first tractable algorithm with such guarantees for this setting. The results are complemented by numerical experiments confirming the validity of this approach.

The talk covers joint work with Felix Krahmer, Mark Iwen and Johannes Maly.

Tuesday, 10.00 - 10.30
Solvable Integration Problems and Optimal Sample Size Selection

ROBERT J. KUNSch
joint work with E. NOvAk and D. Rudolf
Institute for Mathematics, Osnabrück University, Germany, robert.kunsch@uni-osnabrueck.de

A numerical problem is called solvable if for any given error threshold \( \varepsilon > 0 \) and any given confidence level \( 1 - \delta \in (0, 1) \) there exists a randomized algorithm which achieves an error smaller than \( \varepsilon \) at least with probability \( 1 - \delta \) for all problem instances. We call such algorithms \((\varepsilon, \delta)\)-approximating for the respective class of inputs. An algorithm may adapt the sample size to the input based on observations but is required to terminate almost surely.

We provide examples of integration problems which are not solvable. We also discuss a solvable problem where no solution with fixed sample size does exist. Namely, we aim to compute the mean \( \mathbb{E}Y \) of a random variable \( Y \) with unknown statistical dispersion within an absolute error \( \varepsilon \) using i.i.d. observations. Under certain assumptions it is possible to select a sample size based on a variance estimate [1], or, more generally, based on an estimate of the central absolute \( p \)-moment \( \mathbb{E}|Y - \mathbb{E}Y|^p \), with some \( 1 \leq p \leq 2 \). A fixed number of observations is taken for the \( p \)-moment estimate, irrespective of the current input. In fact, this fixed cost is inevitable, the cost for the \( p \)-moment estimate within our algorithm matches a theoretical lower bound for the minimal number of observations needed by any \((\varepsilon, \delta)\)-approximating algorithm with any stopping rule. In addition, the expected cost of our algorithm has the optimal order in \( \varepsilon, \delta \) and the scale of the input.

References


Tuesday, 11.00 - 11.30

Mathematical modeling as an approximation problem

DIRK LANGEMANN
Technische Universität Braunschweig, Institute of Computational Mathematics, AG PDE, Universitätsplatz 2, 38106 Braunschweig, Germany

Mathematical modeling can be seen as the skillful selection of mechanisms, which are describable in mathematical terms and by hands of sufficiently simple mathematics, with the aim to simulate qualitative or quantitative observations from the real world, and to generate predictions about the behaviour of the real world.

There are no tools to answer the question, whether a mathematical model, i.e. the common mathematical description of the selected mechanisms, is suitable for simulating the particular real-world application and for getting predictions about its behaviour. At the same time, the usual intuitive selection of mechanisms based on an established hierarchy and applicability experience fails in life-science applications with their enormous quantitative and qualitative uncertainty about the involved mechanisms. Therefore, the question, whether and in what sense a model can be regarded as good, gets new importance.

The talk presents a formalism which allows to study the modeling process itself by introducing a real-world system, which is assumed to be known. Such a system separates the philosophical-epistemological questions from the mathematics in the modeling process. We will see that the introduction of the hypothetical real-world system opens the view to a range of mathematical questions.

At the same time, this procedure results in the interpretation of mathematical modeling as approximation problem, and a hierarchical model family can be regarded as a set of best-approximating functions in different function spaces. We will use this interpretation to show further analogies between mathematical modeling and approximation theory.

Thursday, 12.00 - 12.30

Condition numbers of Vandermonde matrices with nearly-colliding nodes

DOMINIKA NAGEL
Osnabrück University

The stability of Prony’s method is quite well understood for well-separated nodes on the torus, see e.g. [6, 4, 1]. Here we study the case of nearly-colliding nodes and the condition numbers of associated Vandermonde matrices. The situation with nodes that are lying on a grid has been studied by several authors, see e.g. [2, 3, 5]. Though, in order to apply this to Prony’s method it is necessary to have estimates for condition numbers of Vandermonde matrices with “off-grid” nodes. Starting from the well-separated setting, I present our results for the case when pairs of nodes nearly collide.

Joint work with: Stefan Kunis.

References


Monday, 17.00 - 17.30

**Transformed rank-1 lattices for high-dimensional approximation**

ROBERT NASDALA
*Technische Universität Chemnitz*

We describe an extension of Fourier approximation methods for multivariate functions defined on the torus $\mathbb{T}^d$ to unbounded ones via a multivariate change of coordinate mapping. In this approach we adapt algorithms for the evaluation and reconstruction of multivariate trigonometric polynomials based on single and multiple reconstructing rank-1 lattices and make use of dimension incremental construction methods for sparse frequency sets. Various numerical tests confirm obtained theoretical results for the transformed methods.

Thursday, 17.30 - 18.00

**Parameter tuning for the nonuniform FFT and applications**

FRANZISKA NESTLER
*Technische Universität Chemnitz*

We study the error behavior of the well-known fast Fourier transform for nonequispaced data (NFFT) with respect to the $L_2$-norm. In this context, we compare the resulting approximation errors for different window functions and show that the accuracy may be significantly improved by adapting the shape of the window function. The optimal parameters strongly depend on the involved Fourier coefficients. Finally, we explain the relevance of the suggested parameter tuning strategy for the NFFT based computation of interactions in particle systems.

Thursday, 11.00 - 11.30

**A randomized multivariate matrix pencil method**

THOMAS PETER
*University of Vienna, Faculty of Mathematics*

The matrix pencil method is an eigenvalue based approach for the parameter identification of sparse exponential sums. We derive a reconstruction algorithm for multivariate exponential sums that is based on simultaneous diagonalization. Randomization is used to reduce the simultaneous diagonalization to the eigendecomposition of a single random
matrix. To verify feasibility, the algorithm is applied experimental fluorescence microscopy data.

Thursday, 15.30 - 16.00
More Trouble with Tribbles – Investigation on the Ion Motion Towards Clearing Electrodes in a Particle Accelerator
Gisela Pöplau
Compaec e. G., Rostock, Germany

High brightness beams provided by linac-based accelerators require several measures to preserve their high quality and to avoid instabilities, where the mitigation of the impact of residual ions is one of these measures, in particular if high repetition rates are aimed for.

Over the last decade three ion-clearing strategies: clearing electrodes, bunch gaps and beam shaking have been applied to counteract the degrading impact of the ions on the electron beam. Currently, their merit as clearing strategies for next generation high brightness accelerators such as energy recovery linacs (ERLs) are under intensive investigations with both simulations and measurements.

In this paper, we present numerical studies for the behavior of ions generated by electron bunch passages within the field of electrodes. The objective is to investigate the ion motion towards the electrodes and to study under which circumstances, equilibrium between ion generation and ion-clearing is established. Hereby several ion species and shapes of electrodes are considered with typical parameters of future high current linacs.

Wednesday, 12.00 - 12.30
The cone-beam transform and spherical convolution operators
Michael Quellmalz
Technische Universität Chemnitz

The cone-beam transform consists of integrating a function defined on the three-dimensional space along every ray that starts on a certain scanning set. Based on Grangeat’s formula, Louis [2] states reconstruction formulas based on a new generalized Funk–Radon transform on the sphere.

In this talk, we give a singular value decomposition of this generalized Funk–Radon transform. We use this result to derive a singular value decomposition of the cone-beam transform with sources on the sphere thus generalizing a result of Kazantsev [1].

This is joint work with Ralf Hielscher (Technische Universität Chemnitz) and Alfred K. Louis (Universität des Saarlandes, Saarbrücken).

REFERENCES


Multivariate directional wavelet systems on the torus

KEVIN SCHÖBER
Universität zu Lübeck

In [1, 2] multivariate wavelet systems based on finite dimensional shift-invariant spaces were developed. The corresponding wavelet functions are trigonometric polynomials of de la Vallée Poussin-type, which can be well localized in time and frequency domain. In addition the construction allows for many different dilation matrices on each level including shearing matrices for anisotropic wavelet decompositions.

This talk presents the construction of the trigonometric wavelets such that directional features of functions are revealed in certain wavelet spaces. In particular we are interested in bounds for the wavelet coefficients of cartoon-like functions along curvilinear singularities as they have been proved for discrete shearlet systems on the real line in [3].

REFERENCES


Direct and inverse approximation theorems of functions in the Orlicz type spaces

ANDRII L. SHIDLICH
Institute of Mathematics of NAS of Ukraine

Let $M(t), t \geq 0$, be an Orlicz function, i.e., a non-decreasing convex function such that $M(0) = 0$ and $M(t) \to \infty$ for $t \to \infty$. Let $S_M$ denote the space of all $2\pi$-periodic Lebesgue integrable functions $f \in L$ such that the following quantity is finite:

$$\|f\|_M := \|f\|_{S_M} = \|\{\hat{f}(k)\}_{k \in \mathbb{Z}}\|_{l^1} = \inf \left\{ a > 0 : \sum_{k \in \mathbb{Z}} M(|\hat{f}(k)|/a) \leq 1 \right\},$$

where $\hat{f}(k) := \frac{1}{2\pi} \int_0^{2\pi} f(x)e^{-ikt}dx$, $k \in \mathbb{Z}$, are the Fourier coefficients of $f \in L$.

The quantity

$$E_n(f)_M := \inf_{c_k \in \mathbb{C}} \left\{ \|f - \sum_{|k| \leq n-1} c_k e^{ikx}\|_M \right\}$$

is called the best approximation of $f \in S_M$ by trigonometric polynomials of the order $n - 1$ in the space $S_M$. 

Tuesday, 17.30 - 18.00

Direct and inverse approximation theorems of functions in the Orlicz type spaces
Let $\psi = \{\psi(k)\}_{k=-\infty}^{\infty}$ be a sequence of complex numbers, $\psi(k) \neq 0$. If for the function $f \in L$ with the Fourier series $S[f](x) = \sum_{k \in \mathbb{Z}} \hat{f}(k)e^{ikx}$, the series $\sum_{k \in \mathbb{Z} \setminus \{0\}} \hat{f}(k)e^{ikx}/\psi(k)$ is the Fourier series of a certain function $g \in L$, then $g$ is called (see, for example, [1, Ch. XI]) $\psi$-derivative of the function $f$ and denoted as $g := f^\psi$.

The modulus of smoothness of $f \in S_M$ of index $\alpha > 0$ is defined by

$$\omega_\alpha(f,\delta)_M := \sup_{|h| \leq \delta} \|\Delta^\alpha_h f\|_M = \sup_{|h| \leq \delta} \left\| \sum_{j=0}^{\infty} (-1)^j \binom{\alpha}{j} f(x - jh) \right\|_M, \quad \delta > 0,$$

where $\binom{\alpha}{j} = \alpha(\alpha-1) \ldots (\alpha-j+1)/j!$, $\binom{\alpha}{0} = 1$, $h \in \mathbb{R}$.

One of the results is the following theorem:

**Theorem.** Let $\psi = \{\psi_k\}_{k=-\infty}^{\infty}$ be an arbitrary sequence of complex numbers such that $\psi_k \neq 0$ and $\lim_{|k| \to \infty} |\psi_k| = 0$. If for the function $f \in S_M$ there exists a derivative $f^{(\psi)} \in S_M$, then for any $n \in \mathbb{N}$ and $\alpha > 0$ the following inequalities are true:

$$E_n(f)_M \leq \varepsilon_n E_n(f^{\psi})_M \quad \text{and} \quad E_n(f)_M \leq C \omega_\alpha(f,n^{-1})_M,$$

where $\varepsilon_n = \max_{|k| \geq n} |\psi_k|$ and the constant $C = C(\alpha)$ does not depend on $f$ and $n$.

This is joint work with Stanislav Chaichenko (Donbas State Pedagogical University, Slastiansk, Ukraine) and Fahreddin Abdullayev (Kyrgyz-Turkish Manas University, Bishkek, Kyrgyz Republic; Mersin University, Turkey).


**Tuesday, 17.00 - 17.30**

**Zeros of Polynomials Used in Frequency Analysis**

**Michael Skrzypek**

**Fernuniversität in Hagen**

In the context of the reconstruction of a signal from given samples, the subproblem of determining the frequencies leads to the construction of a polynomial $\rho$. From its zeros the desired frequencies can be obtained by simple computations.

Characteristics of a signal reflect in properties of $\rho$. Depending on them and especially on the location of its zeros, different procedures are available to calculate these ones. In the approach presented here, we factorize $\rho$ as a product of two polynomials, whose zeros are in the interior respectively on the boundary of the (complex) unit circle.

The main focus of this talk is to show how the zeros in the interior of the unit circle can be obtained as eigenvalues of a Hessenberg matrix. Furthermore, we deal with the problem of zeros nearby respectively on the boundary of this circle.
Sparse trigonometric approximation of periodic functions with mixed smoothness

Serhii Stasyuk
Institute of Mathematics, Nat. Acad. Sci. Ukraine, Kyiv

The report is devoted to the exact order bounds of the best \( m \)-term trigonometric approximation (as one of kinds of sparse trigonometric approximation) of Nikolskii–Besov classes \( B^p_{r, \theta} \) of periodic functions with mixed smoothness and others classes (of mixed smoothness) close to \( B^p_{r, \theta} \). Significant attention will be paid to consideration of functions with small mixed smoothness, because the case of small smoothness contain some surprises with respect to the exact order bounds of the mentioned approximation characteristic in comparing with the case of big smoothness. Results presented in the report are published in [1–4].


Uniform polynomial approximation on the sphere

Woula Themistoclakis
C.N.R. National Research Council of Italy, Istituto per le Applicazioni del Calcolo “Mauro Picone”, Napoli, Italy.

The talk concerns the uniform polynomial approximation of a continuous function \( f \) on the Euclidean sphere \( S^q := \{x \in \mathbb{R}^{q+1} : \|x\|_2 = 1\} \), with \( q \geq 2 \). In particular, we focus on the case that sampled values of \( f \) are given at a discrete point set \( X_N \subset S^q \) consisting of nodes of a positive quadrature rule of suitable degree of exactness. In this case hyperinterpolation and filtered hyperinterpolation polynomials can be deduced by discretizing Fourier partial sums and generalized de la Vallée Poussin means (namely delayed weighted mean) of Fourier sums, respectively. These polynomials are based not only on the sampled values of \( f \), but also on the positive quadrature weights that we need to compute if they are not explicitly given. This additional task is not necessary if we consider least squares polynomials and generalized de la Vallée Poussin means of least squares polynomials. We are going to discuss the approximation provided by all the previous polynomials w.r.t. the uniform norm. In particular, a comparable approximation order will be stated for hyperinterpolation and least squares polynomials, as well as for filtered hyperinterpolation polynomials and generalized de la Vallée Poussin means of least squares polynomials.
Thursday, 11.30 - 12.00
Bivariate Prony-type polynomials

HANNA VESLOVSKA
Technische Universität Braunschweig

The parameter estimation of a sum of non-harmonic frequencies is an essential problem in signal processing. Namely, let us consider an $N$-sparse bivariate exponential sum,

$$s_k = \sum_{j=1}^{N} a_j \exp (-i\langle w_j, k \rangle),$$

where $w_j \in (0, 2\pi]^2$, $k \in \mathbb{Z}_+^2$, $\langle w_j, k \rangle$ denotes the inner product of $w_j$ and $k$ and $a_1, a_2, \ldots, a_N \in \mathbb{C}\{0\}$. The aim is to determine the parameters $w_j, j = 1, \ldots, N$, given finitely many samples of $s_k$.

In recent years, a lot of research has been carried out in order to obtain solutions for such problem in multidimensional case (see, e.g., [2]).

Inspired by the one-dimensional approach developed in [1], we propose to find the parameters $w_j, j = 1, \ldots, N$, through common zeros of some special kind of bivariate polynomials. Precisely, using Cantor pairing functions allows us to express bivariate Prony-type polynomials, the polynomials that arise from bivariate Prony’s method, in terms of determinants and to find their exact algebraic representation. As well combining the new approach with an autocorrelation sequence leads to optimistic results regarding stability of solutions in the case of noisy data.

The talk is based on a joint work with J. Prestin from Institute of Mathematics, University of Lübeck.

REFERENCES


Tuesday, 12.00 - 12.30
Combining Adaptive Thinning with Graph Signal Processing

NIKLAS WAGNER
University of Hamburg

In previous work, we have proposed a concept for digital image compression. This compression scheme relies on a locally adaptive algorithm, adaptive thinning, for sparse approximation of images. It utilizes linear splines over anisotropic Delaunay triangulations.

For further improvement on this concept, we want to improve the image quality on some triangles of the triangulation. But this requires a technique which allows us to work with irregular data domains. To this end, we apply signal processing on graphs, which merges algebraic and spectral graph theoretic concepts with computational harmonic analysis.

We interpret the given data as a graph signal on a graph given over each triangle and apply the graph Fourier transform. In the graph spectral domain we are then able to apply filtering methods on the data.

This talk is based on joint work with Armin Iske.
On the regularity of solutions to quasi-linear PDEs

MARKUS WEIMAR
Ruhr-Universität Bochum, Germany

It is well-known that the rate of convergence of numerical schemes which aim to approximate solutions to operator equations is closely related to the maximal regularity of these solutions in certain scales of smoothness spaces of Sobolev and Besov type. For linear elliptic PDEs on Lipschitz domains, a lot of results in this direction already exist. In contrast, it seems that not too much is known for non-linear problems.

In this talk, we are concerned with the $p$-Laplace operator which has a similar model character for quasi-linear equations as the ordinary Laplacian for linear problems. It finds applications in models, e.g., for turbulent flows of a gas in porous media, radiation of heat, as well as in non-Newtonian fluid theory. We discuss a couple of local regularity estimates for the gradient of the unknown solutions. These assertions are then used to derive global smoothness properties by means of wavelet-based proof techniques. Finally, the presented results imply that adaptive algorithms are able to outperform (at least asymptotically) their commonly used counterparts based on uniform refinement.

The material extends assertions which were obtained earlier in joint work with S. Dahlke, L. Diening, C. Hartmann, and B. Scharf [1, 2].

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Acknowledgements  The organizers would like to thank the University of Lübeck and the Fraunhofer Institute ITWM for financial support.

The organizers would like to acknowledge the support by the European Union via the project AMMODIT: H2020-MSCA-RISE-2014 Project number 645672.

Special thanks go to the team of the hotel Schloss Hasenwinkel.