Mecklenburg Workshop

Approximation Methods and Function Spaces

Hasenwinkel

March 16–20, 2015

Organizers
Aicke Hinrichs (Johannes Kepler Universität Linz)
Jürgen Prestin (Universität zu Lübeck)
Tino Ullrich (Universität Bonn)

Dedicated to the 60th birthday of Winfried Sickel

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Program

Monday

	Remise	Schloss
	Chair: J. Prestin	
15.00 - 16.00	**Hans-Jürgen Schmeißer** *Mapping properties of operators in Morrey-type spaces* p. 11	
16.00 - 16.30	Coffee break	
	Chair: L. Pick	
16.30 - 17.00	Vít Musil p. 17 *Optimality of Orlicz domain in Sobolev embeddings*	
17.00 - 17.30	Andrii Shidlich p. 17 *Approximative characteristics of diagonal operators in Orlicz sequence spaces*	
17.30 - 18.00	Sergiy Yanchenko p. 18 *Approximation of the Nikol'skii-Besov classes with dominating mixed smoothness by entire functions of a special form*	

Tuesday

	Remise	Schloss
	Chair: T. Ullrich	
09.00 - 10.00	**Vladimir N. Temlyakov** *Constructive sparse trigonometric approximation and other problems for functions with mixed smoothness* p. 11	
10.00 - 10.30	Dinh Dũng p. 19 *Hyperbolic cross approximation in infinite dimensions*	
10.30 - 11.00	Coffee break	
	Chair: D. Potts	Chair: J. Vybíral
11.00 - 11.30	Lutz Kämmerer p. 19 *Reconstructing multivariate trigonometric polynomials from samples along multiple rank-1 lattices*	Felix Krahmer p. 20 *Compressive sensing with redundant dictionaries and structured measurements*
11.30 - 12.00	Toni Volkmer p. 21 *Fast and stable reconstruction of multivariate high-dimensional algebraic polynomials in Chebyshev form based on samples at rank-1 Chebyshev lattice nodes*	Stefan Loock p. 22 *Near-field phase retrieval with sparsity constraints*
12.00 - 12.30	Béatrice Vedel p. 23 *Hyperbolic wavelet analysis of textures: from anisotropic Besov spaces to applied multifractal analysis*	David James p. 23 *Matrix factorization with binary components*

Tuesday

	Remise	Schloss
	Chair: E. Novak	
14.30 - 15.30	**Stephan Dahlke** p. 12 *The state of the art in Besov regularity for elliptic operator equations*	
15.30 - 16.00	Markus Weimar p. 23 *On optimal wavelet approximations in spaces of Besov type*	
16.00 - 16.30	Coffee break	
	Chair: G. Bourdaud	Chair: F. Filbir
16.30 - 17.00	Markus Hansen p. 24 *New regularity results for elliptic PDEs and application to adaptive approximation methods*	Max Kontak p. 25 *Greedy approximation of probability density functions and its application to the simulation of technical textiles*
17.00 - 17.30	M. Lanza de Christoforis *Composition operators and analyticity of nonlinear integral operators* p. 25	Volker Michel p. 26 *Greedy approximation and regularization in medical imaging and the geosciences*
17.45 - 18.15	Cornelia Schneider p. 27 *Sobolev spaces on Riemannian manifolds with bounded geometry: general coordinates and traces*	Peter Oswald p. 27 *Randomized Schwarz iterations*

Wednesday

	Remise	Schloss
	Chair: H.-J. Schmeißer	
09.00 - 10.00	**Leszek Skrzypczak** p. 12 *Compactness, co-compactness and functions with symmetry conditions*	
10.00 - 10.30	Lenka Slavíková p. 29 *Norms supporting the Lebesgue differentiate theorem*	
10.30 - 11.00	Coffee break	
	Chair: M. Lanza de Christoforis	
11.00 - 11.30	Mikhail L. Goldman p. 29 *Optimal Calderón spaces for Bessel potentials*	
12.00 - 12.30	**Gérard Bourdaud** p. 14 *Composition operators in critical Besov spaces*	
14.30 - 19.30	Excursion to Schwerin and Schwerin Palace p. 10	

Thursday

	Remise	Schloss
	Chair: V. N. Temlyakov	
09.00 - 10.00	**Thomas Kühn** p. 14 *Optimal approximation of multivariate periodic functions*	
		Chair: J. Schneider
10.00 - 10.30	Dauren Bazarkhanov p. 30 *Estimates for some widths of multivariate periodic function classes*	Jörn Schnieder p. 30 *A test for functions to be in the Jacobi-weighted Hardy-space*
10.30 - 11.00	Coffee break	
	Chair: W. Erb	
11.00 - 11.30	Ronny Bergmann p. 31 *The anisotropic periodic Strang-Fix conditions*	Johannes Nagler p. 31 *Lower estimates for positive linear operators*
		Chair: V. Michel
11.30 - 12.00	Ulrich von der Ohe p. 32 *A Prony method for multivariate exponential sums*	Michael Quellmalz p. 33 *Optimal mollifiers for spherical deconvolution*
12.00 - 12.30	Marius Wischerhoff p. 34 *Reconstruction of structured functions from sparse Fourier data*	Katrin Seibert p. 35 *Concentration problem of tensor fields on the sphere*

Thursday

	Remise	Schloss
	Chair: W. Sickel	
14.30 - 15.30	**Lubŏs Pick** p. 15 *Traces of Sobolev functions —old and new*	
15.30 - 16.00	Andreas Seeger p. 35 *Haar bases in Sobolev spaces*	
16.00 - 16.30	Coffee break	
	Chair: M. Hansen	Chair: H. Kempka
16.30 - 17.00	Serhii Stasyuk p. 36 *m-term trigonometric approximation of Nikol'skii-Besov classes*	Jan Vybíral p. 36 *Lorentz spaces of variable integrability*
17.00 - 17.30	Tetiana Lomako p. 36 *Sharp estimates of approximation of functions in Hölder spaces*	Alexander Tyulenev p. 37 *New Besov-type space of variable smoothness and the problem of traces for the weighted Sobolev space*
17.45 - 18.15	Yurii Kolomoitsev p. 37 *Sharp Ulyanov inequalities in quasi-normed spaces*	Dorothee D. Haroske p. 38 *Lipschitz continuity and unboundedness in smoothness Morrey spaces*

Friday

	Remise	Schloss
	Chair: S. Dahlke	
09.00 - 10.00	**Erich Novak** p. 16 *Complexity of oscillatory integrals*	
		Chair: M. Weimar
10.00 - 10.30	Sebastian Mayer p. 39 *Sampling ridge functions on the cube - complexity results*	Robert J. Kunsch p. 39 *Bernstein numbers and the Monte Carlo error*
10.30 - 11.00	Coffee break	
	Chair: A. Hinrichs	
11.00 - 11.30	Mario Ullrich p. 40 *Numerical integration for functions with mixed smoothness*	Van Kien Nguyen p. 40 *Bernstein numbers of embeddings of isotropic Besov spaces*
12.00 - 12.30	**Henryk Woźniakowski** *Tractability of standard and linear information in different settings* p. 16	

Excursion: Schwerin

Schwerin Palace

On Wednesday afternoon, there will be the opportunity to visit the nearby city of Schwerin. Schwerin is the capital of the German state Mecklenburg-Vorpommern. The tour will start with a guided city walk through the old town of Schwerin, which is largely intact thanks to only minor damage in World War II. Highlight of the excursion will be the visit of Schwerin Palace. This castle in romantic style is crowning an island in the Lake Schwerin and houses the parliament of Mecklenburg-Vorpommern.

We leave the hotel for Schwerin at 2:30pm, where we will stay until 6pm or 6:30pm. We arrive back at the hotel at 7:30pm. Costs will be 10 Euro per Person.

Plenary Talks

Monday, 15.00 - 16.00

Mapping properties of operators in Morrey-type spaces

Hans-Jürgen Schmeisser
Friedrich-Schiller-University Jena, Germany

In this talk we deal with vector-valued Morrey-type spaces $\overset{\circ}{L}{}^r_p(\mathbb{R}^n,\,\ell_q)$, $L^r_p(\mathbb{R}^n,\,\ell_q)$ and $H^\rho L_p(\mathbb{R}^n,\,\ell_q)$. The aim is to extend previous results from the scalar case to the vector-valued case. This concerns duality and a transference principle for mapping properties of operators from classical (vector-valued) L_p-spaces to the above Morrey-type spaces. Both topics are closely connected. We show that where $1 < p,\, q < \infty,\ -n/p \le r < 0,\ r + \rho = -n$.
We consider operators T which are assumed to be bounded in $L_p(\mathbb{R}^n)$ and admit an estimate

$$Tf(y) \,\le\, c \int\limits_{\mathbb{R}^n} \frac{|f(z)|}{|y-z|^n}\, dz \qquad \text{for all } \ f \in D(\mathbb{R}^n) \ \text{ and } \ y \notin \operatorname{supp} f.$$

This applies to Calderon-Zygmund type operators, the Hardy-Littlewood maximal function and certain types of Fourier multipliers.
What we are going to present is based on joint work with Marcel Rosenthal (Jena).

Tuesday, 09.00 - 10.00

Constructive sparse trigonometric approximation and other problems for functions with mixed smoothness

V.N. Temlyakov
University of South Carolina; Steklov Institute of Mathematics

Our main interest in this talk is to discuss some approximation problems for classes of functions with mixed smoothness. We discuss the technique, based on a combination of results from hyperbolic cross approximation and recent results on greedy approximation, which was used to obtain sharp estimates for best m-term approximation with respect to the trigonometric system. We give some observations on numerical integration and approximate recovery of functions with mixed smoothness. We present lower bounds, which show that one cannot improve accuracy of sparse grids methods with $\asymp 2^n n^{d-1}$ points in the grid by adding 2^n arbitrary points. In case of numerical integration these lower bounds provide best known lower bounds for optimal cubature formulas and for sparse grids based cubature formulas.

Tuesday, 14.30 - 15.30

The state of the art in Besov regularity for elliptic operator equations

Stephan Dahlke
Philipps–University of Marburg

We will be concerned with the regularity of the solutions to elliptic operator equations in Lipschitz domains contained in $\mathbf{R}^d$. In particular, we will consider the specific scale $B^s_\tau(L_\tau), 1/\tau = s/d + 1/p$, of Besov spaces. The regularity of the solutions in these Besov spaces determines the order of approximation that can be achieved by adaptive and other nonlinear numerical schemes. In contrast, the approximation order of nonadaptive (uniform) schemes is determined by the classical L_p-Sobolev scale. Recent studies have shown that under very natural assumptions the solutions to elliptic operator equations are much smoother in the specific Besov scale than in the L_p-Sobolev scale which justifies the use of adaptive schemes. The aim of this talk is to give an overview on the current state of the art. Special emphasis will be laid on results obtained in cooperation with Winfried Sickel.

Wednesday, 09.00 - 10.00

Compactness, co-compactness and functions with symmetry conditions

Leszek Skrzypczak
Adam Mickiewicz University, Poznań, Poland

The interplay between regularity and decay of functions that satisfy certain symmetry conditions, e.g. of radial functions was observe in late 60-ties of last century by several people working in PDE. In particular W.Strauss proved that the following inequality:

$$|x|^{\frac{n}{2}-1}|f(x)| \ \leq \ \|\nabla f\|_2$$

holds for radial function defined on $\mathbb{R}^n$, $n \geq 2$. It was also noticed that this inequality implies the compactness of Sobolev embeddings of subspaces of inhomogeneous Sobolev spaces consisted of radial functions. One can also observe that the lack of compactness of the usual Sobolev embeddings on $\mathbb{R}^n$ is also related to the action of some group of isometries on function spaces. This lead to the concept of co-compact embeddings and profile decompositions. Apart of the radial functions one can consider the weaker symmetry conditions, e.g. so called block-radial functions. The function defined on $\mathbb{R}^n$ is block-radial if it is invariant with respect to the action of the following group $SO(n_1) \times \ldots \times SO(n_k)$, $n_1 + \ldots n_k = n$ and $n_i \geq 2$.

In the talk I will present the survey of results proved with Winfried Sickel and Cyril Tintarev (Uppsala) that related to the Strauss inequality and the compactness phenomena. In particular we generalized the Strauss inequality and look for the sufficient and necessary condition for the inequality to hold. The conditions are formulated in terms of the radial subspaces of Besov-Lizorkin-Triebel spaces. Moreover we look for the counterpart of the inequality for the block-radial functions.

References

[1] W. Sickel and L. Skrzypczak, *Radial subspaces of Besov and Lizorkin-Triebel spaces: extended Strauss lemma and compactness of embeddings,* J. Fourier Anal. Appl. **106** (2000), 639-662.

[2] L. Skrzypczak, *Rotation invariant subspaces of Besov and Triebel-Lizorkin space: compactness of embeddings, smoothness and decay of functions*, Rev. Mat. Iberoamericana, **1**8 (2002), 267-299.

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Wednesday, 11.30 - 12.30

Composition operators in critical Besov spaces

Gérard Bourdaud
Université Paris Diderot

We deal with the composition operators $T_f(g) := f \circ g$ acting on vector valued Besov and Lizorkin-Triebel spaces. Let k, n be natural numbers such that $1 \le k \le n$, let $p, q \in [1, +\infty]$, and let $s = n/p > 1$. If f is a function of $\mathbb{R}^k$ to $\mathbb{R}$ such that T_f takes $B^s_{p,q}(\mathbb{R}^n, \mathbb{R}^k)$ to $B^s_{p,q}(\mathbb{R}^n)$, then ∇f belongs locally uniformly to $B^{s-1}_{p,q}(\mathbb{R}^k, \mathbb{R}^k)$. A similar statement holds by replacing $B^s_{p,q}$ by $F^s_{p,q}$.

Thursday, 09.00 - 10.00

Optimal approximation of multivariate periodic functions

Thomas Kühn
Universität Leipzig

In this talk I will present some recent joint results with Winfried Sickel on optimal linear approximation of multivariate functions on the d-dimensional torus $\mathbb{T}^d$. Further co-authors of the papers that I will discuss are Fernando Cobos (Madrid) and Tino Ullrich (Bonn).

The functions under consideration belong to isotropic Sobolev spaces $H^s(\mathbb{T}^d)$ of fractional smoothness $s > 0$ and to Sobolev spaces $H^s_{\text{mix}}(\mathbb{T}^d)$ with dominating mixed derivatives, and the error is measured either in the L_2-norm or in the L_∞-norm. For any fixed dimension $d \in \mathbb{N}$ and arbitrary smoothness $s > 0$, the asymptotic behaviour of the approximation numbers of the corresponding embeddings is well known. In the isotropic case, for example, one has

$$c_s(d)\, n^{-s/d} \le a_n(I_d : H^s(\mathbb{T}^d) \to L_2(\mathbb{T}^d)) \le C_s(d)\, n^{-s/d} \tag{1}$$

for all $n \in \mathbb{N}$ with certain constants $c_s(d)$ and $C_s(d)$. However, in the literature very little is known about these constants. Note that $n^{-s/d}$ is almost constant for small n, say for $1 \le n \le 2^d$, while 2^d might already beyond the reach in numerical computations, if d is large. Hence, without further information on the constants, the inequalities (1) are useless for practical purposes.

Our main focus is on the dependence of the constants $c_s(d)$ and $C_s(d)$ on the dimension d (and the smoothness s), with respect to a family of natural norms. We show that, for fixed $s > 0$ and $d \in \mathbb{N}$, the limit

$$\lim_{n\to\infty} n^{s/d}\, a_n(I_d : H^s(\mathbb{T}^d) \to L_2(\mathbb{T}^d))$$

exists, and we determine its exact value. It turns out that it decays polynomially in d, where the exponent depends on the chosen norm. Moreover, we give matching two-sided estimates for small n, i.e. in the preasymptotic range, where the asymptotic is not yet visible. Similar results are obtained for the mixed spaces $H^s_{\mathrm{mix}}(\mathbb{T}^d)$, again for a variety of equivalent natural norms.

Moreover we consider a wider class of Sobolev-type spaces on $\mathbb{T}^d$, whose norms are weighted ℓ_2-sums of Fourier coefficients. For such spaces we derive a general method that allows to translate estimates for L_2-approximation into L_∞-results, this relation might be of independent interest. In the proof we use functional-analytic tools, in particular 2-summing operators.

In the language of information-based complexity, the n-th approximation number $a_n(I_d)$ is nothing but the worst-case error of linear algorithms that use at most n arbitrary linear informations. I will mention some consequences of our result, in terms of tractability of approximation problems. Finally I state some open questions.

Thursday, 14.30 - 15.30

Traces of Sobolev functions—old and new

Luboš Pick

Charles University in Prague

The talk will focus on the classical problem of traces of functions from Sobolev spaces, which had originated in connection with some specific problems in PDEs and then mushroomed into a separate field of research in functional analysis and the function spaces theory. One important property enjoyed by functions from the Sobolev space $W^{m,p}(\mathbb{R}^n)$, where $m \in \mathbb{N}$ and $p \in [1, \infty]$, is that their restrictions, called traces, to lower dimensional spaces $\mathbb{R}^d$ can be properly defined, provided that the dimension d of the relevant subspaces is not too small, depending on the values of n, m and p. In such case one can ask whether some properties such as a certain degree of integrability of a trace can be expected, and, naturally, which of these properties are the best possible. We shall survey both classical and recent results concerning traces of Sobolev functions. We shall consider basic questions concerning the very existence of trace as well as deeper problems such as optimal trace embeddings involving specific function spaces.

Friday, 09.00 - 10.00

Complexity of oscillatory integrals

Erich Novak
University of Jena (FSU)

We analyze univariate oscillatory integrals for integrands from the standard Sobolev space H^s or for the space C^s, where s is a natural number. We find bounds for the worst case error of optimal algorithms that use n function values.

This lecture is based on a paper in the J. Complexity (2015) and on a recent unpublished manuscript with Mario Ullrich, Henryk Woźniakowski and Shun Zhang.

Friday, 11.30 - 12.30

Tractability of Standard and Linear Information in Different Settings

Henryk Woźniakowski
Columbia University and University of Warsaw

Tractability of multivariate problems has been analyzed for two classes of information used by algorithms. For *linear* information we allow to use arbitrary linear functionals, whereas for *standard* information we allow to use only function values. Obviously, linear information is at least as powerful as standard information. One of the main problem of tractability studies is to verify when the power of standard and linear information is roughly the same. In this talk we present a number of results comparing the power of standard and linear information in the worst case, average case and randomized settings. This talk will be based on join work with Erich Novak.

Session Talks

Monday

Monday, 16.30 - 17.00 (Remise)

Optimality of Orlicz domain in Sobolev embeddings

Vít Musil
Charles University in Prague

Given a rearrangement-invariant Banach function space $Y(\Omega)$, we consider the problem of the existence of an optimal (largest) domain Orlicz space $L^A(\Omega)$ satisfying the Sobolev embedding $W^m L^A(\Omega) \hookrightarrow Y(\Omega)$. We present a complete solution of this problem within the class of Marcinkiewicz endpoint spaces which covers several important examples. It turns out that the answer is negative for these examples and it suggests that this tool is rather strong in negative cases. We also prove a certain sufficient condition for a positive result with no restriction to target spaces.

Monday, 17.00 - 17.30 (Remise)

Approximative characteristics of diagonal operators in Orlicz sequence spaces

Andrii Shidlich[1], Stanislav Chaichenko[2]

[1]*Institute of Mathematics of NASU, Kyiv, Ukraine;*
[2]*Donbas State Pedagogical University, Slavyansk, Ukraine*

Let $M(t)$, $t \geq 0$, be an Orlicz function, that is a non-decreasing convex down function such that $M(0) = 0$ and $M(t) \to \infty$ as $t \to \infty$. Orlicz sequence space l_M, defined by the function $M(t)$, is the linear space of all sequences $x = \{x_k\}_{k=1}^{\infty}$ of real numbers such that $\sum_k M(x_k) < \infty$. Equipped with the norm

$$||\mathbf{x}||_{l_M} := \inf\{\alpha > 0 : \sum_{k=1}^{\infty} M(|x_k|/\alpha) \leq 1\}$$

it is a Banach space. Let $\lambda = \{\lambda_k\}_{k=1}^{\infty}$ be an arbitrary bounded sequence of the positive numbers and let $T : x = \{x_k\}_{k=1}^{\infty} \to Tx = \{\lambda_k x_k\}_{k=1}^{\infty}$ be a diagonal operator defined on the space l_M. Following S.B. Stechkin, for any sequence $x \in l_M$, consider the quantity $\sigma_n(x, l_M)$ of its best n-terms approximation, which is given by the relation

$$\sigma_n(x, l_M) := \inf_{\gamma_n} E_{\gamma_n}(x, l_M) = \inf_{a_i, \gamma_n} ||x - P_{\gamma_n}||_{l_M} = \inf_{a_i, \gamma_n} ||x - \textstyle\sum_{i \in \gamma_n} a_i e_i||_{l_M},$$

where γ_n are the arbitrary collections of n positive integers and $a_i \in \mathbb{R}$.

Theorem. Assume that $0 < p < \infty$ is an arbitrary positive number and $M(t)$ is the Orlicz function such that $M(t^{1/p})$ is also the Orlicz function. Let also $\lambda = \{\lambda_k\}_{k=1}^{\infty}$ be an arbitrary non-increasing sequence of the positive numbers, satisfying condition $\lim_{k\to\infty} \lambda_k = 0$. Then for any $n \in \mathbb{N}$, the following equality is true:

$$\begin{aligned}\sigma_n(T : l_p \to l_M) &:= \sup_{x\in Bl_p} \sigma_n(Tx, l_M) \\ &= \sup_{s>n} (\textstyle\sum_{k=1}^{s} \lambda_k^{-p})^{-\frac{1}{p}} \big(M^{-1}(1/(s-n))\big)^{-1},\end{aligned} \tag{1}$$

where M^{-1} is the inverse function of M. The least upper bound on the right-hand side of (1) is attained at some finite value s^* of s.

Monday, 17.30 - 18.00 (Remise)

Approximation of the Nilol'skii–Besov classes with dominating mixed smoothness by entire functions of a special form

Sergiy Yanchenko
Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine

In the present talk, we discuss the problem of the approximation of the classes $S^{\boldsymbol{r}}_{p,\theta}B(\mathbb{R}^d)$ [1] of functions of many variables which are called Nikol'skii–Besov classes.

Consider a set $\mathfrak{M}$ formed by the vectors $\boldsymbol{s} = (s_1, \dots, s_d)$, $s_j \in \mathbb{Z}_+$, $j = \overline{1, d}$, and $Q_{2^{\boldsymbol{s}}} := \big\{\lambda \in \mathbb{R}^d : \ \eta(s_j)2^{s_j-1} \leqslant |\lambda_j| < 2^{s_j}, \quad j = \overline{1, d}\big\}$, where $\eta(0) = 0$ and $\eta(t) = 1$, $t > 0$.

For $f \in L_q(\mathbb{R}^d)$, $1 < q < \infty$, we set $S^{\mathfrak{M}}(f, \boldsymbol{x}) = \sum_{\boldsymbol{s}\in\mathfrak{M}} \delta^*_{\boldsymbol{s}}(f, \boldsymbol{x})$, where $\delta^*_{\boldsymbol{s}}(f, \boldsymbol{x}) = = \mathfrak{F}^{-1}(\chi_{Q_{2^{\boldsymbol{s}}}} \cdot \mathfrak{F}f)$, $\chi_{Q_{2^{\boldsymbol{s}}}}$ is the characteristic function of the set $Q_{2^{\boldsymbol{s}}}$, $\mathfrak{F}f$ and $\mathfrak{F}^{-1}f$ respectively direct and the inverse Fourier transform of the function f.

Further, for $f \in L_q(\mathbb{R}^d)$ we consider the approximative characteristic

$$e^{\mathfrak{F}}_M(f)_q = \inf_{\mathfrak{M}:\mathrm{mes}\bigcup_{\boldsymbol{s}\in\mathfrak{M}} Q_{2^{\boldsymbol{s}}}\leqslant M} \|f(\cdot) - S^{\mathfrak{M}}(f, \cdot)\|_q,$$

where mesA is the Lebesgue measure of the set A.

Theorem. Let $1 < p < \infty$, $r_1 > \frac{1}{p}$. Then for $1 \leqslant \theta \leqslant \infty$ the order relation is true

$$e^{\mathfrak{F}}_M(S^{\boldsymbol{r}}_{p,\theta}B)_\infty = \sup_{f\in S^{\boldsymbol{r}}_{p,\theta}B} e^{\mathfrak{F}}_M(f)_\infty \asymp \big(M^{-1}\log^{\nu-1} M\big)^{r_1-\frac{1}{p}}\big(\log^{\nu-1} M\big)^{1-\frac{1}{\theta}}.$$

References

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Tuesday

Tuesday, 10.00 - 10.30 (Remise)

Hyperbolic cross approximation in infinite dimensions

Dinh Dũng
Vietnam National University, Hanoi

We give tight upper and lower bounds of the cardinality of the index sets of certain hyperbolic crosses which reflect mixed Sobolev-Korobov-type smoothness and mixed Sobolev-analytic-type smoothness in the infinite-dimensional case where specific summability properties of the smoothness indices are fulfilled. These estimates are then applied to the linear approximation of functions from the associated spaces in terms of the ε-dimension of their unit balls. Here, the approximation is based on linear information. Such function spaces appear for example for the solution of parametric and stochastic PDEs. The obtained upper and lower bounds of the approximation error as well as of the associated ε-complexities are completely independent of any dimension. Moreover, the rates are independent of the parameters which define the smoothness properties of the infinite-variate parametric or stochastic part of the solution. These parameters are only contained in the order constants. This way, linear approximation theory becomes possible in the infinite-dimensional case and corresponding infinite-dimensional problems get tractable. This is joint work with M. Griebel (Bonn).

Tuesday, 11.00 - 11.30 (Remise)

Reconstructing multivariate trigonometric polynomials from samples along multiple rank-1 lattices

Lutz Kämmerer
Technische Universität Chemnitz

The approximation of periodic functions of dominating mixed smoothness using as single rank-1 lattice as spatial discretization leads to a non-optimal number of used sampling values.

In order to reduce the number of needed sampling values, we consider multiple rank-1 lattices as spatial discretizations. Motivated by the concepts

of sparse grids and rank-1 lattices as spatial discretizations for multivariate trigonometric polynomials, we develop an algorithm that constructs a spatial discretization of multivariate trigonometric polynomials that allows a unique reconstruction of the trigonometric polynomials. In addition, we present a corresponding high dimensional fast Fourier transform, that allows the evaluation and the reconstruction of the multivariate trigonometric polynomials using the new discretizations. Various numerical tests promise a nearly optimal number of sampling values, small condition numbers of the discrete Fourier transform, and low computational costs of the construction of multiple rank-1 lattices as well as of the fast Fourier transform.

Tuesday, 11.00 - 11.30 (Schloss)

Compressive sensing with redundant dictionaries and structured measurements

Felix Krahmer
University of Göttingen

Consider the problem of recovering an unknown signal from undersampled measurements, given the knowledge that the signal has a sparse representation in a specified dictionary D. This problem is now understood to be well-posed and efficiently solvable under suitable assumptions on the measurements and dictionary, if the number of measurements scales roughly with the sparsity level. One sufficient condition for such is the D-restricted isometry property (D-RIP), which asks that the sampling matrix approximately preserve the norm of all signals which are sufficiently sparse in D. While many classes of random matrices are known to satisfy such conditions, such matrices are not representative of the structural constraints imposed by practical sensing systems. We close this gap in the theory by demonstrating that one can subsample a fixed orthogonal matrix in such a way that the D-RIP will hold, provided this basis is sufficiently incoherent with the sparsifying dictionary D. We also extend this analysis to allow for weighted sparse expansions. Consequently, we arrive at compressive sensing recovery guarantees for structured measurements and redundant dictionaries.

This is joint work with Deanna Needell and Rachel Ward.

Tuesday, 11.30 - 12.00 (Remise)

Fast and stable reconstruction of multivariate high-dimensional algebraic polynomials in Chebyshev form based on samples at rank-1 Chebyshev lattice nodes

Toni Volkmer
TU Chemnitz

We consider multivariate high-dimensional algebraic polynomials in Chebyshev form, i.e.,

$$p(\boldsymbol{x}) = \sum_{\boldsymbol{k}\in I} a_{\boldsymbol{k}}\, T_{\boldsymbol{k}}(\boldsymbol{x}) = \sum_{\boldsymbol{k}\in I} a_{\boldsymbol{k}} \prod_{t=1}^{d} T_{k_t}(x_t), \quad a_{\boldsymbol{k}} \in \mathbb{R}, \tag{0.1}$$

where $I \subset \mathbb{N}_0^d$ is a non-negative index set of finite cardinality, $d \in \mathbb{N}$ is the dimensionality and $T_{\boldsymbol{k}} : [-1,1]^d \to [-1,1]$, $T_{\boldsymbol{k}}(\boldsymbol{x}) := \prod_{t=1}^{d} T_{k_t}(x_t)$, are multivariate Chebyshev polynomials for $\boldsymbol{k} := (k_1, \ldots, k_d)^\top \in \mathbb{N}_0^d$ built from a tensor product of Chebyshev polynomials of the first kind $T_k : [-1,1] \to [-1,1]$, $T_k(x) := \cos(k \arccos x)$, $k \in \mathbb{N}_0$. For example, if the index set $I = \{\boldsymbol{k} \in \mathbb{N}_0^d \colon \|\boldsymbol{k}\|_1 \leq n\}$, $n \in \mathbb{N}_0$, is the ℓ_1-ball, then any algebraic polynomial of (total) degree $\leq n$ in d variables restricted to the domain $[-1,1]^d$ can be represented by (0.1). In this talk, we describe a fast method for the evaluation of an arbitrary polynomial p from (0.1) at the nodes of an arbitrary d-dimensional rank-1 Chebyshev lattice

$$\tilde{\Lambda}(\boldsymbol{z}, M) := \left\{ \boldsymbol{x}_j := \cos\left(\frac{j}{M}\pi \boldsymbol{z}\right) : j = 0, \ldots, M \right\},$$

which is characterized by the generating vector $\boldsymbol{z} \in \mathbb{N}_0^d$ and the size parameter $M \in \mathbb{N}_0$. We discuss conditions on rank-1 Chebyshev lattices $\tilde{\Lambda}(\boldsymbol{z}, M)$, such that the exact reconstruction of polynomials p from (0.1) based on samples $p(\boldsymbol{x}_j)$, $j = 0, \ldots, M$, is possible with an arbitrarily chosen index set $I \subset \mathbb{N}_0^d$ of finite cardinality. For building such a rank-1 Chebyshev lattice $\tilde{\Lambda}(\boldsymbol{z}, M)$ suitable for reconstruction, we present an algorithm which is based on a component-by-component approach and we show numerical results. We give a method for the fast, exact and stable reconstruction of the coefficients $a_{\boldsymbol{k}}$, $\boldsymbol{k} \in I$, from the samples $p(\boldsymbol{x}_j)$, $j = 0, \ldots, M$. Moreover, we discuss relations to Padua points and we point out relations to the approximation on hyperbolic cross index sets.

Both for the evaluation and reconstruction of an arbitrary polynomial p from (0.1) with given index set $I \subset \mathbb{N}_0^d$ of finite cardinality at rank-1 Chebyshev lattice nodes, we only apply a single one-dimensional discrete/fast cosine transform of type I with length M and we compute some simple index transforms, yielding an arithmetic complexity of $\mathcal{O}(M \log M + d\, 2^d |I|)$ arithmetic operations.

This is joint work with Daniel Potts.

Tuesday, 11.30 - 12.00 (Schloss)

Near-field phase retrieval with sparsity constraints

Stefan Loock
Universität Göttingen

We propose the application of sparsity constraints for the problem of phase retrieval. In nanoscale photonic imaging the wave propagation in the near-field can be described by the Fresnel propagator

$$[\mathcal{D}_\tau f](\xi) = \frac{1}{\tau^2} \int_{\mathbb{R}^2} f(x) \exp\left(\frac{\mathrm{i}\pi}{\tau^2} \|x - \xi\|^2 \right) \mathrm{d}x$$

for some experimental parameter $\tau > 0$. Unfortunately, reconstructing the phase of $\mathcal{D}_\tau f$ from measurements $M = |\mathcal{D}_\tau f|$ is a non-linear, ill-posed inverse problem.

The underlying assumption is that the function $f \in L^2(\mathbb{R}^2)$ models a cartoon-like image and therefore can be well approximated by directional wavelet frames such as curvelets or shearlets. We will present a method based on generalized projection algorithms to solve the inverse problem of object reconstruction. We will use ℓ_1-minimization of shearlet coefficients and projections onto the set of valid measurements.

Since experimental data is corrupted by Poisson noise, we will use a relaxation/regularization of the Douglas-Rachford algorithm known as relaxed averaged alternating reflections. We will present numerical simulations to compare our approach to existing methods.

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Tuesday, 12.00 - 12.30 (Remise)

Hyperbolic wavelet analysis of textures : from anisotropic Besov spaces to applied multifractal analysis

Beatrice Vedel
Université de Bretagne Sud, Vannes

Joint work with P. Abry, M. Clausel, S. Jaffard, S. Roux

I will first show that the Hyperbolic wavelet basis provide an almost exact characterization of classical and anisotropic Besov spaces.

Using this 'universal basis' which allow to characterize any anisotropy, we are able to define a multifractal formalism which can be use to classify and/or characterize anisotropic textures.

Finally I will show numerical experiments on a class of anisotropic gaussian textures, that show that the hyperbolic wavelet analysis allows to estimate both regularity and anisotropy of the texture.

Tuesday, 12.00 -12.30 (Schloss)

Matrix factorization with binary components

David James
University of Göttingen

Motivated by an application in computational biology, we consider low-rank matrix factorization with $\{0, 1\}$-constraints on the first of the factors and optionally convex constraints on the second one. Despite apparent intractability, we present an algorithm that provably recovers the underlying factorization, provided there exists a unique solution. We will further show that by choosing a suitable random model for the binary factor, there will always be a unique solution with high probability. This is joint work with Felix Krahmer, Martin Slawski, Matthias Hein and Pavlo Lutsik.

Tuesday, 15.30 - 16.00 (Remise)

On optimal wavelet approximations in spaces of Besov type

Markus Weimar
Philipps-University Marburg, Faculty of Mathematics and Computer Science, Workgroup Numerics and Optimization

This talk is concerned with the approximation of embeddings between Besov-type spaces defined on bounded multidimensional domains or (patchwise smooth) manifolds. We compare the quality of approximation of three different strategies based on wavelet expansions. For this purpose, sharp rates of convergence corresponding to classical uniform refinement, best N-term, and best N-term tree approximation will be presented. In particular, we will see that whenever the embedding of interest is compact greedy tree approximation schemes are as powerful as abstract best N-term approximation and that (for a large range of parameters) they can outperform uniform schemes based on a priori fixed (hence non-adaptively chosen) subspaces. This observation justifies the use of adaptive non-linear algorithms in computational practice, e.g., for the approximate solution of boundary integral equations arising from physical applications.

If time permits, implications for the related concept of approximation spaces associated to the three approximation strategies will be discussed.

The results to be presented are work in progress within the framework of the DFG-Project “Adaptive Wavelet and Frame Techniques for Acoustic BEM” (DA 360/19-1).

Tuesday, 16.30 - 17.00 (Remise)

New regularity results for elliptic PDEs and application to adaptive approximation methods

Markus Hansen
TU München

We present some new results on the regularity of solutions to elliptic PDEs in Lipschitz domains with polyhedral structure. More precisely, these solutions belong to certain weighted Sobolev spaces, which in turn are embedded into Besov or Triebel-Lizorkin spaces. Both scales are known to be closely related to Approximation spaces w.r.t. adaptive approximation methods (n-term wavelet approximation and adaptive Finite element methods).

These regularity results are subsequently combined with approximation results for these adaptive methods to derive convergence rates for adaptive methods for elliptic PDEs. As a second application, the previous results are utilized for the approximation of parametric problems and elliptic problems with random coefficients.

Tuesday, 16.30 - 17.00 (Schloss)

Greedy approximation of probability density functions and its application to the simulation of technical textiles

Max Kontak
Geomathematics Group, University of Siegen, Germany

Estimating the probability density function (PDF) of a continuous random variable based on a given sample is a common task in statistics. If no a-priori knowledge about the PDF is available, it is a standard approach to use so-called kernel density estimators (KDEs), which possess various advantageous properties. However, if the sample size is large, it can be computationally expensive to evaluate a KDE, which is, e. g., necessary for sampling from the estimated distribution.

Therefore, a greedy algorithm to estimate PDFs is developed, which yields an approximation that needs a reduced number of kernel evaluations. Furthermore, this algorithm is not restricted to a fixed kernel type, as KDEs are. Instead, arbitrary functions can be mixed, e. g., kernels of different types, polynomials, or wavelets.

This algorithm is applied to a sample of points on the 2-sphere, which represent directions of fibers inside a real technical textile, obtained by a CT scan. Finally, an outlook is given on the opportunity to use the estimated PDF for the simulation of these technical textiles.

Tuesday, 17.00 - 17.30 (Remise)

Composition operators and analyticity of nonlinear integral operators

Massimo Lanza de Cristoforis
University of Padua, Italy

We prove an analyticity theorem for a class of nonlinear integral operators which involve a Nemytskij type composition operator. Such operators appear in the applications as pull-backs of layer potential operators.

To do so, we resort to a general result on composition operators acting in Banach algebras and we split our operators into nonlinear integral operators acting in Roumieu classes and composition operators.

The talk is based on joint work with Paolo Musolino.

Tuesday, 17.00 - 17.30 (Schloss)

Greedy approximation and regularization in medical imaging and the geosciences

Volker Michel
Geomathematics Group, University of Siegen, Germany

Greedy algorithms and, in particular, different kinds of matching pursuits for the efficient approximation of unknown functions have intensively been investigated for at least two decades. In many cases, seen in an abstract and broader sense, the interpolation or approximation of signals which are given via samples plays a dominant role. However, the concept of a matching pursuit is also interesting for the regularization of ill-posed inverse problems. For this reason, the Geomathematics Group Siegen has developed different variants of a matching pursuit (FMP, RFMP, ROFMP) which are also applicable to the case where a function f is unknown and values of linear and continuous functionals $\mathcal{F}^k f = y_k$, $k = 1, \ldots, l$ are unknown. This more general concept includes the cases of samples of f ($\mathcal{F}^k f := f(x_k)$) as well as, e.g., samples of the right-hand side of an inverse problem $Tf = g$, $\mathcal{F}^k f := g(z_k)$. The particular features of the new methods are

- Data and solution can be elements of completely separate spaces.
- The objective function of the matching pursuit includes a penalty term for the regularization of unstable problems.
- The algorithms allow the combination of different types of trial functions (e.g. orthogonal polynomials together with spline/wavelet basis functions) such that the obtained approximation inherits the advantages of each of these basis systems.
- The algorithm produces an iterative sequence of stable approximations which converges to the solution of the Tikhonov-regularized normal equation (or the Moore-Penrose inverse in the non-regularized case).
- An analogue of the Orthogonal Matching Pursuit allows a sparser representation of the constructed approximation.

We are particularly interested in applications in the geosciences and in medical imaging and, therefore, focus on the implementation of the algorithms with dictionaries of functions on the sphere or the ball.
In this talk, theoretical results and some examples of numerical results for practical applications are summarized.

Tuesday, 17.45 - 18.15 (Remise)

Sobolev spaces on Riemannian manifolds with bounded geometry: general coordinates and traces

CORNELIA SCHNEIDER
Friedrich-Alexander-University Erlangen-Nürnberg

We study fractional Sobolev and Besov spaces on noncompact Riemannian manifolds with bounded geometry. Usually, these spaces are defined via geodesic normal coordinates which, depending on the problem at hand, may often not be the best choice. We consider a more general definition subject to different local coordinates and give sufficient conditions on the corresponding coordinates resulting in equivalent norms. Our main application is the computation of traces on submanifolds with the help of Fermi coordinates.

Tuesday, 17.45 - 18.15 (Schloss)

Randomized Schwarz iterations

PETER OSWALD
Jacobs University Bremen

The name Schwarz iterative methods has been coined in the late 1980ies as a theoretical framework for investigating domain decomposition and multilevel methods for variational problems. They are based on the notion of stable space splittings of a Hilbert space H into a family of auxiliary Hilbert spaces H_i, each equipped with its own variational subproblem, and in essence represent a more constructive version of the method of alternating projections. In this talk, we consider the multiplicative Schwarz iteration

$$u^{j+1} = u^j + \omega_j w_i^j, \qquad (0.2)$$

where finding the update direction w_i^j involves solving the i-th subproblem, and ω_j is a relaxation parameter. Which $i = i_j$ is chosen in the j-th update step matters, until recently only deterministic orderings have been considered.

The recent interest in investigating random (and greedy) orderings was triggered by a 2009 paper by Strohmer and Vershynin on the convergence of a randomized Kaczmarz method which received much attention, both due to the simplicity of the error analysis and the sometimes improved performance. In [1], we generalized their result to the setting of Schwarz iterative methods with finitely many subproblems. Even though the a priori bounds for greedy and random orderings are identical, the numerical tests reported in [1] show that the greedy version leads to faster convergence in practice, and that combinations of randomization and greedy approaches can remedy slow convergence of the cheaper randomized iteration. A complementary estimate was proved in [2] for a block version of (0.2) with random orderings.

Finally, (0.2) can be executed also for infinite space splittings, where it turns from an iterative solver into an approximation algorithm for the solution u of the underlying variational problem by finite linear combinations of subproblem solutions. In [3], convergence of a modified version of (0.2) for both greedy and random orderings has been established, together with quantitative convergence rates under additional assumptions for u. The greedy case is a direct generalization of earlier work by Temlyakov et al. on greedy algorithms in Hilbert spaces.

References

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Wednesday

Wednesday, 10.00 - 10.30 (Remise)

Norms supporting the Lebesgue differentiation theorem

Lenka Slavíková
Charles University in Prague

We analyze a version of the Lebesgue differentiation theorem, where Lebesgue norms are replaced with more general rearrangement-invariant norms. We establish necessary and sufficient conditions for a norm of this kind to support the Lebesgue differentiation theorem. In particular, we characterize Lorentz and Orlicz norms for which this theorem holds.

This is a joint work with P. Cavaliere, A. Cianchi and L. Pick.

Wednesday, 11.00 - 11.30 (Remise)

Optimal Calderón spaces for Bessel potentials

Mikhail L. Goldman
Peoples' Friendship University of Russia, Moscow, Russia

The talk is devoted to generalized Bessel potentials constructed by the convolutions of generalized Bessel-McDonald kernels with functions from a basic rearrangement invariant space. If the criterion for the embedding of potentials into the space of bounded continuous functions is satisfied, we establish an equivalent description for the cones of the moduli of continuity of potentials in the uniform norm. This enables us to obtain a criterion for the embedding of potentials into the Calderón space.
In the case of generalized Bessel potentials constructed over a basic weighted Lorentz space we can describe explicitly the optimal Calderón space for such an embedding. The main results are joint work with Dorothee D. Haroske (Jena) and were published in [1-3].

References

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[3] M.L. Goldman and D.D. Haroske. Optimal Calderón Space for Bessel Potentials. *Dokl. Akad. Nauk*, 458(5):510–513, 2014. Russian; English transl.: Dokl. Math. 90, 599–602 (2014).

Thursday

Thursday, 10.00 - 10.30 (Remise)

Estimates for some widths of multivariate periodic function classes

Dauren Bazarkhanov
Institute of Mathematics and Math Modeling

We will give some new estimates (sharp in order) for linear and Fourier widths of some multi–parameter classes of periodic functions in the space $L_p(\mathbb{T}^d)$, $1 < p < \infty$.

Thursday, 10.00 - 10.30 (Schloss)

A test for functions to be in the Jacobi-weighted Hardy-space

Jörn Schnieder
Lübeck

In [2], we constructed an unconditional polynomial Schauder-basis $(p_{\alpha,\beta,n})_{n\in\mathbb{N}_0}$ of optimal degree with Jacobi-orthogonality for the weighted Hardy-space $H^1_{\alpha,\beta}$ and the spaces $L^p_{\alpha,\beta}$, $1 < p < \infty$. In this talk we want to show that we can give a test, in terms of the behavior of the Fourier-$(p_{\alpha,\beta,n})$ series of a function $f(x)$, for $f(x)$ to belong to $H^1_{\alpha,\beta}$. In particular, we sketch the proof for the following theorem:

Theorem. A necessary and sufficient condition for a series

$$\sum_{n=0}^{\infty} c_n p_{\alpha,\beta,n}(x)$$

to converge unconditionally in $L^1_{\alpha,\beta}$ is that its sum $f(x)$ belongs to $H^1_{\alpha,\beta}$.

The idea of the proof is to combine refined asymptotic estimates from [2] and [3], atomic decomposition methods for weighted Hardy spaces and proof structures from the works of Wojtaszczyk and Woźniakowski [4] as well as Chang and Ciesielski [1] (rooting back to the famous H^1-results from Carleson).

References

[1] Chang, S.-Y., A., Ciesielski, Z. Spline characterisations of H^1. *Stud Math. 75* (1983), 183 - 192.

[2] Erb, W., Schnieder, J. Unconditional polynomial Schauder Basis with Jacobi-weights. *submitted* (2015).

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Thursday, 11.00 - 11.30 (Remise)

The anisotropic periodic Strang-Fix conditions

Ronny Bergmann
Technische Universität Kaiserslautern

For error estimates of an approximation by equidistant translates of a multivariate periodic function the periodic Strang-Fix conditions enable a unified investigation. For these conditions the error is derived for functions having a certain decay of Fourier coefficients. Recently, many approaches such as curvelets or shearlets focus on certain directions when analyzing or decomposing multivariate functions.

In this talk, we first generalize the periodic Strang-Fix conditions to an anisotropic version in order to also take directions into account. Using these elliptic periodic Strang-Fix conditions we look at the multivariate interpolation of periodic functions on arbitrary lattices, i.e., patterns they form on the d-dimensional torus.

We further introduce spaces and norms, where each function is of different directional smoothness due to the imposed anisotropic decay of the Fourier coefficients. Sampling these functions on corresponding lattices and employing the periodic anisotropic Strang-Fix conditions, we derive an upper bound for the error of interpolation by translates of a function. The interpolant especially takes the different directional smoothness properties into account.

Thursday, 11.00 - 11.30 (Schloss)

Lower estimates for positive linear operators

Johannes Nagler
Universität Passau

The aim of this talk is to show a general functional analysis based framework to prove lower estimates of positive linear operators for the approximation error by moduli of smothness or K-functionals using spectral properties. To this end, we consider positive linear operators on a general Banach function space with smooth range.

There are mainly two conditions on T to derive lower estimates. The technique used in the proof uses the convergence of the iterates of the operator. Furthermore, if the limiting operator is annihilated by a differential operator then a lower estimate can be stated. We will show first that the iterates of every positive linear operator of finite-rank having a partition of unity property converges by spectral properties. Then we will generalize our setting to so called quasi-compact operators, where all limiting points in the spectrum are inside the unit ball. This setting guarantees that the fixed point space of the operator and its adjoint is in fact finite-dimensional. Provided that bases for these spaces are known, we show how to obtain the limiting operator of the iterates. This result is constructive in the sense that the limit operator can be explicitly calculated by the inverse of a Grammian matrix.

Thursday, 11.30 - 12.00 (Remise)

A Prony method for multivariate exponential sums

Ulrich von der Ohe
Osnabrück University

The parameter reconstruction of an exponential sum

$$f(x) = \sum_{j=1}^{M} c_j \exp(\mathrm{i}\omega_j x), \quad c_j \in \mathbf{C}, \quad \omega_j \in [0, 2\pi[,$$

given a finite number of samples $(x, f(x))$, $x \in \mathbf{R}^d$, is a classical problem. This has already been considered and essentially solved in 1795 by de Prony [4]. With his method the parameters c_j and ω_j are reconstructed separately and the involved linear recurrence relations are exploited to construct a polynomial whose zeros are precisely the values $\exp(\mathrm{i}\omega_j)$, $j = 1, \ldots, M$. Recently, variants and generalizations of this approach have been studied, among others by Potts-Tasche [8], Plonka-Wischerhoff [7], Peter-Plonka [6], Filbir-Mhaskar-Prestin [5], Beylkin-Monzón [2], Andersson-Carlsson-de Hoop [1], and Candès-Fernandez-Granda [3].

Using a combination of analytic and algebraic arguments, we propose a generalization to d-variate exponential sums

$$f(x) = \sum_{j=1}^{M} c_j \exp(\mathrm{i}\langle \omega_j, x\rangle), \quad c_j \in \mathbf{C}, \quad \omega_j \in [0, 2\pi[^d,$$

where the parameters are realized as common zeros of a system of d-variate polynomials.

This is joint work with Stephan Kunis, Thomas Peter, and Tim Römer.

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Thursday, 11.30 -12.00 (Schloss)

Optimal mollifiers for spherical deconvolution

Michael Quellmalz
TU Chemnitz

A famous problem in tomography was described by Funk [1], who considered the reconstruction of a function defined on the two-dimensional sphere from its integrals along all great circles of the sphere. This problem became known as the inversion of the Funk–Radon transform. The Funk–Radon transform, together with e.g. the spherical sine and cosine transforms, is a spherical convolution operator, which can be expressed as a multiplication operator in the space of Fourier coefficients with respect to spherical harmonics.

The subject of this talk is the inversion of convolution operators, called deconvolution. In most applications, the given function is measured only at finitely many points. In addition, these measurements are usually disrupted by some noise. Additionally, deconvolution is an ill-posed problem. A common regularization scheme is the so-called mollifier method, which means that the reconstructed function is smoothened by convoluting it with a mollifier. There are several suggestions for mollifiers around, cf. [2, 3].

We have asked ourselves how an optimal mollifier could look like. We have assumed that the function we want to reconstruct has a certain degree of smoothness with respect to Sobolev spaces. Then we could show that there exists a class of mollifiers that minimizes the expected L^2–error amongst all mollifiers, asymptotically as the number of available measurements grows. The optimality of these mollifiers is illustrated by numerical tests.

References

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Thursday, 12.00 - 12.30 (Remise)

Reconstruction of structured functions from sparse Fourier data

Marius Wischerhoff

Georg-August-Universität Göttingen

In several scientific areas, such as radio astronomy, computed tomography, and magnetic resonance imaging, the reconstruction of structured functions from the knowledge of samples of their Fourier transform is a common problem. For the analysis of the examined object, it is important to reconstruct the underlying original signal as exactly as possible.

In this talk, we aim to uniquely recover structured functions from only a small number of Fourier samples. For this purpose, the Prony method, which is a deterministic method for the recovery of sparse trigonometric functions, is used as key instrument to derive algorithms for unique recovery by means of a smallest possible set of Fourier data.

We will give an overview of reconstruction results for different function classes, and we will consider two classes in detail. First, we will examine linear combinations of N non-uniform shifts of a given bivariate function. Here, the unknown shift parameters and corresponding coefficients in the linear combination are recovered from sparse Fourier data. Unique recovery of the parameters is possible by using only $3N+1$ Fourier samples on three lines through the origin. For this purpose, two predetermined lines are considered, while the third sampling line is chosen dependently on the results obtained by employing the samples from the first two lines. These results are compared to the case when only predetermined sampling lines are used. The presented approach can be generalized to the case of d-variate functions with $d > 2$.

Secondly, we turn to the reconstruction of polygonal shapes in the real plane. Here, a convex or non-convex polygonal domain D with N vertices is considered. It is shown that the vertices and their order can be reconstructed by taking $3N$ samples of the Fourier transform of the characteristic function of the polygonal domain D. Again, two predetermined sampling lines and an appropriately chosen third line are considered.

Thursday, 12.00 - 12.30 (Schloss)

Concentration problem of tensor fields on the sphere

Katrin Seibert
Geomathematics Group, University of Siegen, Germany

Spherical Slepian functions (by Simons et al.) form a complete orthonormal basis on the unit sphere. They are a basis transformation of the spherical harmonics. Indeed, they have the advantage of the optimal concentration within a region of interest and are, thus, relevant for local problems. Recently, vector Slepian functions have also been constructed (by Plattner and Simons) for the analysis of geomagnetic data. Satellite gravity gradiometry data (SGG data) like the data recorded by the satellite mission GOCE is given in the form of tensors. So, an extension of the method of Slepian functions to tensors is useful. However, tensor Slepian functions on the sphere have only rarely been investigated so far. We construct a method for an efficient calculation of such local trial functions. They base not only on the tensor spherical harmonics (by Freeden and Schreiner) but also on the spin-weighted spherical harmonics (by Newman and Penrose).

Thursday, 15.30 - 16.00 (Remise)

Haar bases in Sobolev spaces

Andreas Seeger
University of Wisconsin-Madison

We determine on which Sobolev and Triebel-Lizorkin spaces the Haar system is an unconditional basis and obtain related upper and lower bounds for various Haar projection numbers. This is joint work with Tino Ullrich (Bonn).

Thursday, 16.30 - 17.00 (Remise)

m-term trigonometric approximation of Nikol'skii-Besov classes

Serhii Stasyuk
Institute of Mathematics of the National Academy of Sciences of Ukraine

Investigations are devoted to finding the exact order estimates of m-term trigonometric approximations of Nikol'skii-Besov classes of periodic functions of several variables. In particular, Nikol'skii-Besov classes are determined through a complete modulus of smoothness or through a mixed modulus of smoothness. We devote special attention for the case of small smoothness, when we establish the exact order estimates of the best m-term trigonometric approximations of mentioned classes.

Thursday, 16.30 -17.00 (Schloss)

Lorentz spaces of variable integrability

Jan Vybíral
Charles University Prague, Czech Republic

We combine the concept of Lorentz spaces with function spaces of variable integrability. The classical Lorentz spaces are typically defined in terms of non-increasing rearrangement and function spaces of variable smoothness are not rearrangement invariant, which poses several obstacles already in the definition of the new scale of function spaces. We discuss also the Marcinkiewicz interpolation theorem in the frame of these new spaces and answer in negative the question of Diening, Hästö and Nekvinda if there is an easy counterpart for this kind of interpolation for spaces of variable smoothness.

This is a joint work with Henning Kempka (TU Chemnitz).

Thursday, 17.00 - 17.30 (Remise)

Sharp estimates of approximation of functions in Hölder spaces

Tetiana Lomako
Institute of Mathematics, NAS of Ukraine, Kiev

We study sharp estimates of approximation of functions by trigonometric and algebraic polynomials in the Hölder spaces $H_p^{r,\alpha}$ for all $0 < p \leq \infty$ and $0 < \alpha \leq r$. By using modifications of the classical moduli of smoothness, we give improvements of the direct and inverse theorems of approximation and prove the criteria of the precise order of decrease for the best approximation in $H_p^{r,\alpha}$. In addition, strong converse inequalities for general methods of approximation of functions are obtained.

It is joint work with Yurii Kolomoitsev (Institute of Mathematics, NAS of Ukraine, Kiev) and Jürgen Prestin (Universität zu Lübeck, Institut für Mathematik).

Thursday, 17.00 - 17.30 (Schloss)

New Besov-type space of variable smoothness and the problem of traces for the weighted Sobolev space

Alexander Tyulenev
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For the weighted Sobolev space $W_p^l(\mathbb{R}^n,\gamma)$ a complete description of the trace spaces for planes of dimension $1 \le d < n$ is obtained. The weight γ depends on all variables and locally satisfies the Muckenhoupt condition. It appears that in the case $1 \le r < p < \infty$ the trace space for $W_p^l(\mathbb{R}^n,\gamma)$, $\gamma \in A_{\frac{p}{r}}^{loc}(\mathbb{R}^n)$ is the Besov type space $\widetilde{B}_{p,p,r}^l(\mathbb{R}^d,\{\gamma_k\})$ with variable smoothness $\{\gamma_k\}$. The norm in the space $\widetilde{B}_{p,q,r}^l(\mathbb{R}^d,\{\gamma_k\})$ is defined with the help of local best approximations in the L_r - metric.

Various properties of the space $\widetilde{B}_{p,q,r}^l(\mathbb{R}^d,\{\gamma_k\})$ are studied by using the method of nonlinear spline approximation for all values of the parameters $0 < p, q, r < \infty$, $l \in \mathbb{N}$ under the minimal assumptions on the variable smoothness $\{\gamma_k\}$. For example we present the atomic decomposition theorem, embedding theorems and description of the trace space of $\widetilde{B}_{p,q,r}^l(\mathbb{R}^d,\{\gamma_k\})$. The space $\widetilde{B}_{p,q,r}^l(\mathbb{R}^d,\{\gamma_k\})$ is compared with 2-microlocal Besov space $B_{p,q}^{\{\gamma_k\}}(\mathbb{R}^d)$ intensively studied by many mathematicians.

Thursday, 17.45 - 18.15 (Remise)

Sharp Ulyanov inequalities in quasi-normed spaces

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Let $f \in L_p(\mathbb{T})$, where $0 < p \le \infty$ and $\mathbb{T} = [0, 2\pi)$. The modulus of smoothness of order $\beta > 0$ is defined by

$$\omega_\beta(f,h)_p = \sup_{|\delta|<h} \left\| \sum_{\nu=0}^{\infty} (-1)^\nu \frac{\beta(\beta-1)\dots(\beta-\nu+1)}{\nu!} f(x+\nu\delta) \right\|_{L_p(\mathbb{T})}.$$

We prove the following sharp Ulyanov inequality.

Theorem. *Let* $0 < p \le 1$, $p < q \le \infty$, $\alpha \in \mathbb{N} \cup (1/q_1 - 1, \infty)$, $\gamma \ge 0$, *and* $\alpha + \gamma \in \mathbb{N} \cup (1/p - 1, \infty)$.

(A) *Let* $f \in L_p(\mathbb{T})$. *Then for any* $\delta \in (0, 1]$ *and* $m \ge \gamma$ *the following inequality holds*

$$\omega_\alpha(f, \delta)_q \le C \left(\frac{\omega_{\alpha+\gamma}(f, \delta)_p}{\delta^\gamma} \sigma\left(\frac{1}{\delta}\right) + \left(\int_0^\delta \left(\frac{\omega_{\alpha+m}(f, t)_p}{t^{\frac{1}{p}-\frac{1}{q}}} \right)^{q_1} \frac{dt}{t} \right)^{\frac{1}{q_1}} \right), \quad (1)$$

where

$$\sigma(t) = \begin{cases} t^{\frac{1}{p}-1}, & \gamma > 1 - \frac{1}{q_+}, \\ t^{\frac{1}{p}-1} \ln^{\frac{1}{q}} 2t, & 0 < \gamma = 1 - \frac{1}{q_+}, \\ t^{\frac{1}{p}-\frac{1}{q}-\gamma}, & 0 \le \gamma < 1 - \frac{1}{q_+}, \end{cases}$$

$q_1 = \min(q, 1)$, $q_+ = \max(q, 1)$, *and* C *is a constant independent of* δ *and* f.

(B) *Inequality (1) is sharp in the following sense. There exists a function* $f_0 \in L_q(\mathbb{T})$, $f_0 \not\equiv \mathrm{const}$, *such that for* $m > \gamma + 1/p - 1$ *and* $\delta \to 0$ *the left-hand side is equivalent to the right-hand side of the inequality.*

In this talk, we will discuss some generalizations of inequality (1) to different realizations of K-functionals in the case of several variables.

This is joint work with Sergey Tikhonov (ICREA, Centre de Recerca Matemàtica, Barcelona).

Thursday, 17.45 - 18.15 (Schloss)

Lipschitz continuity and unboundedness in Smoothness Morrey Spaces

Dorothee D. Haroske
Friedrich-Schiller-University Jena, Germany

We consider different scales of smoothness spaces of Morrey type and determine their growth and continuity envelopes. In particular, regarding unboundedness of such distributions, some specific behaviour appears which is different from the 'classical' situation in Besov or Lizorkin-Triebel spaces.

This is joint work with S. Moura (Coimbra), L. Skrzypczak (Poznań), D. Yang (Beijing) and W. Yuan (Beijing).

Friday

Friday, 10.00 - 10.30 (Remise)

Sampling ridge functions on the cube - complexity results

SEBASTIAN MAYER
University of Bonn

We present some work in progress concerning the approximation of ridge functions defined on the high-dimensional cube $[-1,1]^d$. In general, the problem suffers from the curse of dimensionality when function values are the only information at hand. This holds true even for randomized algorithms. However, by extending a recent algorithm of Cohen et al., we can prove weak tractability if additional compressibility information about the ridge direction is available. This is joint work with Benjamin Doerr (Ecole Polytechnique, Paris) and Daniel Rudolph (U Jena).

Friday, 10.00 - 10.30 (Schloss)

Bernstein numbers and the Monte Carlo error

ROBERT J. KUNSCH
FSU Jena

We are interested in lower bounds for the approximation of linear operators between Banach spaces with algorithms that may use at most n arbitrary linear functionals as information. Lower error bounds for deterministic algorithms can easily be found by *Bernstein widths*. A similar connection between *Bernstein numbers* and lower bounds for the Monte Carlo approximation of operators between *arbitrary* Banach spaces was found recently and will be presented in this talk. As usual, for the proof we switch to an average case setting. Namely, we follow an approach due to Heinrich (1992) [1] which is based on truncated Gaussian measures. The existence of a suitable Gaussian distribution follows from Lewis' Theorem.
As an application, we find that for the L_∞ approximation of smooth functions from the class $C^\infty([0,1]^d)$ with uniformly bounded partial derivatives, randomized algorithms suffer from the curse of dimensionality, as it is known for deterministic algorithms.

REFERENCES

[1] S. Heinrich. Lower Bounds for the Complexity of Monte Carlo Function Approximation. *Journal of Complexity*, 8:277–300, 1992.

Friday, 11.00 - 11.30 (Remise)

Numerical integration of functions with mixed smoothness

MARIO ULLRICH
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We prove upper bounds on the order of convergence of Frolov's cubature formula for numerical integration in function spaces of dominating mixed smoothness on the unit cube with homogeneous boundary condition. More precisely, we study worst-case integration errors for Besov $\mathbf{B}^s_{p,\theta}$ and Triebel-Lizorkin spaces $\mathbf{F}^s_{p,\theta}$ and our results treat the whole range of admissible parameters ($s > 1/p$). In particular, we treat the case of small smoothness which is given for Triebel-Lizorkin spaces $\mathbf{F}^s_{p,\theta}$ in case $1 < \theta < p < \infty$ with $1/p < s \leq 1/\theta$. The worst-case error in these cases shows a completely different behavior compared to "large" smoothness $s > 1/\theta$. Moreover, we present modifications of the algorithm which lead to the same bounds also for spaces on the cube without homogeneous boundary condition.
(Joint work with Tino Ullrich, HCM Bonn)

Friday, 11.00 - 11.30 (Schloss)

Bernstein numbers of embeddings of isotropic Besov spaces

VAN KIEN NGUYEN
Friedrich-Schiller University Jena, Germany

Bernstein numbers are considered in Approximation and Operator Theory. Let T be a bounded linear operator from a Banach space X into a Banach space Y. Then the n-th Bernstein number of T is defined as

$$b_n(T : X \to Y) = \sup_{L_n} \inf_{\substack{x \in L_n \\ \|x\|=1}} \|Tx|Y\|$$

where the supremum is taken over all subspaces L_n of X with dimension n. In this talk we shall give two-sided sharp estimates of Bernstein numbers of the embeddings from Besov spaces $B^t_{p_1,q}((0,1)^d)$ into $L_{p_2}((0,1)^d)$.

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