

# Third Conference Mathematics for Life Sciences

Rivne, September 15 - 19, 2015

#### Information on the project:

The goal of the EUMLS project is to contribute to overcoming the historical communication and cross-disciplinary barriers that exist between the biosciences and mathematics through a comprehensive 48 month research staff exchange programme between five Ukrainian mathematical institutes and three partners in Germany, Italy, and Norway already active in different aspects of computational life sciences.

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Information on the workshop. This is the annual conference to be organized in the framework of the Marie Curie IRSES project "EU-Ukrainian Mathematicians for Life Sciences" (EUMLS) and of the Marie Curie RISE project "Approximation Methods for Modelling and Diagnosis Tools" (AMMODIT).

The previous conferences were in Kyiv, September 2012, and in Olenivka, September 2013.

The conference is devoted to recent research in life sciences based on applications of mathematics to biological and medical studies. It is a multidisciplinary meeting forum for researchers who develop and apply mathematical and computational tools to the study of phenomena in the broad fields of biology, ecology, medicine, bioengineering, environmental science, etc.

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Dates: 15–19 September, 2015. Location: Rivne, Ukraine.

#### ABSTRACTS

#### A. ANTONIOUK <sup>1</sup>

# Least action principle for weighted porous media equation

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In the beginning 18th century Leibniz, Maupertuis, Euler claimed that all physical phenomenons may be obtained from the Least Action Principle and since Lagrange and Hamilton it was well understood for the classical mechanics. However only in 1966 V.I. Arnold in [2] achieved it for the fluid dynamics. To do this he remarked that the group of volume preserving diffeomorphisms  $\mathcal{D}_{\mu}(M)$  of a manifold M ( $\mu$  being a given volume element on M) is the appropriate configuration space for the hydrodynamics of an incompressible fluid. In this framework the solutions to the Euler equation become geodesic curves with respect to the right invariant metric on  $\mathcal{D}_{\mu}$ .

The main our result [1] shows that the weighted porous media equation ([3], [4]), which generalizes the standard porous media equation,

$$\frac{\partial u}{\partial t} = \left(-u \cdot \nabla + \frac{1}{2}\Delta\right) \left(\|u\|^{q-2}u\right) + \nabla P$$

may be also obtained in the framework of Least Action Principle for specially chosen energy functional. In the particular case of q=2 this recovers the Navier-Stokes equation.

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### Mathematical model of immune response with the influence of external factors

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- G.I. Marchuk proposed his mathematical model of immune reaction [1] in the eighties of past century. The simplest version of the model describes humoral response, with the assumption of constant environmental conditions implying constant parameters of the model.

The first attempt to model the influence of external conditions could be found in [2], where periodic seasons are reflected by periodic model coefficients.

We investigate the influence of environmental pollution on the course of immune reaction described by the Marchuk model, that is we consider mathematical model that reads

$$\dot{V} = \beta (1 - \delta V^n) V - \gamma V F,$$

$$\dot{E} = r (1 - E_{\Delta}) E,$$

$$\dot{m} = \sigma V - \mu_m m + \mu_2 (E - 1),$$

$$\dot{C} = \alpha \xi(m) V_{\tau} F_{\tau} - \mu (C - C^*) - \mu_1 (E - 1),$$

$$\dot{F} = \rho C - \eta \gamma F V - \mu_f F,$$
(1)

where V(t), C(t) and F(t) reflect the amount of antigen, plasma cells and antibodies, respectively, m(t) is the characteristic of organ-target,  $\tau$  is the delay of immune response reflecting the time needed to form the cascade of plasma cells, E(t) is the average indicator of pollution with the delay  $\Delta$  characterising the average recovery time of ecological balance.

For the model described by Eqs. (1), stationary solutions were found, their stability was investigated and numeric modelling was held.

 G. I. Marchuk, Math. Modell. Immun. Respon. Infect. Diseas., Springer-Verlag, New York, 1997.

#### V. V. BILET <sup>1</sup>, O. A. DOVGOSHEY <sup>2</sup>

### Local strong porosity and its application to the theory of pretangent spaces

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A pretangent space to the metric space X at a point p is a set of equivalence classes of sequences, tending to a sequence (p,p,...p,...) with a given rate [1]. Let  $E\subseteq [0,\infty)$ . Write  $\tilde{E}_0^d$  for the set of sequences  $\tilde{\tau}=\{\tau_n\}_{n\in\mathbb{N}}$  such that  $\tau_{n+1}\leq \tau_n, \lim_{n\to\infty}\tau_n=0$  and  $\tau_n\in E\setminus\{0\}$  for every  $n\in\mathbb{N}$ .

**Definition.** Let  $E \subseteq [0,\infty)$  and  $\tilde{\tau} \in \tilde{E}_0^d$ . The set E is  $\tilde{\tau}$ -strongly porous at 0 if there are a constant  $c \geq 1$  and a sequence of intervals  $(a_n,b_n) \subset [0,\infty) \setminus E$  such that  $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$  and  $\frac{1}{c}a_n \leq \tau_n \leq ca_n$  for every  $n \in \mathbb{N}$ .

If for every sequence  $\tilde{\tau} \in \tilde{E}_0^d$  there is a subsequence  $\tilde{\tau}' = \{\tau_{n_k}\}_{k \in \mathbb{N}}$  for which the set E is  $\tilde{\tau}'$ -strongly porous at 0, then the set E is w-strongly porous at 0.

**Theorem.** [2] Let (X, d, p) be a pointed metric space. All pretangent spaces to X at p are bounded if and only if the set  $\{d(x, p) : x \in X\}$  is w-strongly porous at 0.

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### Numerical methods of complex analysis for solving problems of electrical impedance tomography

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We propose the method and the appropriate algorithm of identification (reconstruction) of the conductivity coefficient according data of tomography of the applied potentials. This approach is based on numerical methods of complex analysis: methods of quasiconformal mappings combined with finite difference methods, the summary representations methods of G. M. Polozhii, the domain decomposition alternating method of Schwarz, methods of the block iterations and methods for solving ill-posed problems of A. N. Tikhonov. See [1, 1, 2, 3, 5].

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A. Ya. BOMBA <sup>1</sup>, O. V. PRYSIAZHNIUK <sup>1</sup>

### Modeling of singularly perturbed processes of heat-mass transfer in nanoporous environments

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Modeling of heat and mass transfer in nanoporous environments is a promising area of research on the use of the filter with nanoporous filling for cleaning polluted streams. Now a lot of publications are devoted to modeling of adsorption of pollution in nanoporous materials [1, 2]. However the question of consideration of all components of process heat and mass transfer in these environments for predict the effectiveness of the cleaning devices that combine adsorptive, chemical and thermal processes, remains relevant. Since the diffusion and mass transfer composition of the relevant processes are small as compared to convection, corresponding problems which described heat and mass transfer in nanoporous media are singularly perturbed, i.e. they contain small coefficients at some members. The solutions of these problems are found as asymptotic series in [3].

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A. Ya. BOMBA <sup>1</sup>, A. P. SAFONYK <sup>2</sup>,

## Simulation of biological treatment of water in aeration tanks regenerator

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Today, by reducing water quality in open water problem appears to reduce human pressure on water objects. One way is to increase the efficiency of water treatment facilities. Currently the most common is biological treatment of wastewater, due to its versatility and low cost of operation. The efficiency of biological treatment plants wastewater can be markedly improved by optimizing technical modes of biological wastewater treatment. The actual process for this is computer simulation of biological treatment of impurities in water purification systems and determine the parameters that most influence the time and quality of treatment.

We construct the mathematical model, which describes the patterns of change of concentrations flakes, dispersed bacteria, autolysis products and substrate in the regenerator aeration tank (i.e. in the first phase of the cleaning process). We find the solution of the corresponding model problem with function pdepe (Matlab). We calculate the concentration distribution of contamination and bacteria during cleaning fluid.

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### A. Y. BOMBA $^1,$ Y. V. TURBAL $^2,$ O. V. RADOVENIUK $^2,$ M. Y. TURBAL $^2$

# The Markov moment problem and algorithms of the solitary waves trajectories identification in continuous media

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In recent years the investigation of separated waves plays an important role in many applied scientific fields. Travelling wave solutions

can describe various phenomena in fluid mechanics, hydrodynamics, optics, plasma physics, solid state physics, biology, meteorology, and other fields. In this paper we consider some approaches to the problem of solitary waves of trajectory identification in continuous media. In the case of solitary wave trajectory function

$$r(t) = \alpha_1 exp(-\xi_1 \phi(t)) + \alpha_2 exp(-\xi_2 \phi(t)) + \dots + \alpha_m exp(-\xi \phi(t))$$
 (1)

problem of trajectory identification is reduced to checking the compatibility of the system of nonlinear equations:

$$\alpha_1 exp(-\xi_1 \phi_i) + \alpha_2 exp(-\xi_2 \phi_i) + \dots + \alpha_m exp(-\xi_m \phi_i) = r_i, \quad (2)$$

i=1,2,...,k, where  $\alpha_1,\xi_1,\alpha_2,\xi_2,...\alpha_m,\xi_m$  are unknown parameters,  $\phi_i,r_i$  are given values. To solve the exponential moment problem which is determined by the system (2), we reduce (2) to the classical case of the Chebyshev system  $F=(1,t,t^2,...,t^m)$ .

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A. Ya. BOMBA <sup>1</sup>, S. V. YAROSCHAK <sup>1</sup>, A. M. SINCHUK <sup>1</sup>

### Numerical method quasiconformal mapping modeling filtration processes for the effect of hydraulic fracturing cracks

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A new methodology for modelling of one- and multiphase filtration in porous and permeable difficult (shale) oil layers with influence of hydraulic fracturing cracks and numerical algorithms for solving the appropriate boundary problems are developed in [1-4]. On the basis of which, new software implementation for calculation of filtration and capacitive characteristics are created.

Approach to simulation of non-isothermal process of the displacement in the elements of areal flooding disturbance of filtration flow by the cracks fracturing under conditions of thermal mode is developed. Corresponding numerical algorithm is constructed; numerical computations are made; data analysis and results are calculated.

It is generalized solution methodology of two-dimensional marginal tasks of one-phase filtration in permeable difficult in case of spatial-deviated deposits with hydraulic fracturing cracks and contiguous deformation processes in nearfield zone of deposit, when the investigated process is described under condition of quasistationary and filtrational flow by specially modified Darcy law in relation to the critical value of the pressure gradient.

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#### D. A. CHUPIS $^{1,2}$

# Liquid velocity measurement in the channel with diameter typical for blood vessels

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Mathematical simulations methods are widely used for the vessels blood flows investigations with the case of pathologies. Indirect noninvasive methods used for the simulation results verification such as vessel field of velocity visualization by means of the ultrasonic diagnostics, magnetic resonance tomography, angiography are distorted over the information-measuring systems disturbances influence and can't clearly give the right view about the simulation initial conditions right choice.

Device for the environment motion characteristics direct investigations in the channel with typical size comparable to the blood vessel is presented. Miniature measuring transducer with the 0,1 mm sensor size is designed for the high resolution providing within the measurements carrying out. Precious coordinate device for sensor positioning inside the vessel is proposed. Results of the designed measuring tool prior experimental investigations are given.

#### N. V. DEREVIANKO <sup>1</sup>

# Approximation of the classes of periodic functions of several variables by some linear methods

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In this talk we deal with the classes  $H_p^\Omega$  of periodic functions of several variables. These classes are in some sense generalization of the Nikol'skii classes  $H_p^r$ . The aim was to extend some previous results obtained by A. V. Andrianov, V. N. Temlyakov [1] from the classes  $H_p^r$  to the classes  $H_p^\Omega$ .

We established the upper order estimates for the approximation of functions of the classes  $H_p^{\Omega}$  by some linear methods constructed by using operators of special form in the space  $L_p$  for  $1 \leq p \leq \infty$ . Using this result we obtained the exact order estimates of orthoprojective widths of the classes  $H_p^{\Omega}$  in the space  $L_p$  for  $p \in \{1, \infty\}$  [2].

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# On spaces extremal for the Gomory-Hu inequality

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Let (X,d) be a finite nonempty ultrametric space. In 1961 E. C. Gomory and T. C. Hu [1] proved the inequality  $|\operatorname{Sp}(X)| \leqslant |X|$  where  $\operatorname{Sp}(X) = \{d(x,y) \colon x,y \in X\}$ . Using weighted Hamiltonian cycles and weighted Hamiltonian paths we give new necessary and sufficient conditions under which the Gomory-Hu inequality becomes an equality. We find the number of non-isometric (X,d) satisfying the equality  $|\operatorname{Sp}(X)| = |X|$  for given  $\operatorname{Sp}(X)$ . Moreover it is shown that every finite semimetric space Z is an image under a composition of mappings  $f \colon X \to Y$  and  $g \colon Y \to Z$  such that X and Y are finite ultrametric space, X satisfies the above equality, f is an  $\varepsilon$ -isometry with an arbitrary  $\varepsilon > 0$ , and g is a ball-preserving mapping.

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#### V. A. DMYTRUK <sup>1,2</sup>

# Computer simulation of anesthetic propagation during epidural anesthesia

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<sup>2</sup> Lviv Polytechnic National University, Ukraine E-mail: Veronika.A.Dmytruk@lpnu.ua A mathematical model of the anesthetic solution transferring for a patient's epidural anesthesia after its injection through a catheter into the epidural space of the spinal cord is considered.

By means of the method of solving contact-boundary value problems of convection-diffusion in binary regular structures using integral transforms for the spatial variables separately in contacting domains [1], exact solutions are obtained for two contact-boundary value problems defined within this model.

On the basis of the obtained solutions, a software is developed for computer simulation of mass transfer of the solution for epidural anesthesia through the intervertebral holes to determine a necessary dose of this solution, its component composition, the ratio of its volume to the concentration depending on the age of the patient to achieve an adequate level of anesthesia.

It is established that for the elderly patients, i.e. when the convection diffusion rate of the drug is decreased, the dose of anesthetic should be reduced accordingly.

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Yu. Yu. FEDCHUN <sup>1</sup>, V. I. GERASIMENKO <sup>2</sup>

### Kinetic equations of soft active matter

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We develop an approach to the description of collective behavior of large number of interacting constituents of mathematical biology within the framework of the evolution of marginal observable of these systems.

Using suggested approach, we derive the non-Markovian kinetic equation for a system of interacting stochastic Markovian processes, modeling the evolution of soft active matter on the microscopic scale. For such

system we establish the kinetic equation and prove the property of the propagation of initial chaos in mean field scaling limit.

We also construct the Vlasov-type kinetic equation with initial correlations, characterizing the condensed states of interacting living cells, and the property of the propagation of initial correlations is established.

The obtained results we applied to the problem of the description of the typical hemokinetic properties of blood flows.

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#### N. B. FILIMONOVA <sup>1</sup>

### Wavelet-transformation of EEG based on the Krawtchouk functions

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Wavelet transformation is employed to investigate the nonstationarity EEG signals. Technology of wavelet is based on the generalized representation of a signal as the weighed sum of the base functions (mother-wavelet) which are multiplied on the features of decomposed of a signal using this mother-wavelet. As mother-wavelet we used the Krawtchouk's functions. The obtained Krawtchouk's wavelet coefficients are sufficient in determining the time-localization and scale (frequency) structure of EEG. The use of the Krawtchouk functions allows adequately recognizes and recreates the narrow impulsive constituents. The invariance property in the wavelet-transformation allows us not to care of the fixed initial point of signal. In addition during the process of artifacts identification this method allows to find them in the window of analysis. Picking up and deleting of artifacts are the necessary stage of EEG preparation for

further wavelet-analysis. In the classic spectral analysis there are artifacts related, for example, with blinking of eyes (oculography artifacts) or other muscular motions, are marked by hand as artifacts and willfully removed from further analysis. In case of wavelet-analysis it is necessary to save a dynamics and attachment of EEG to the corresponding functional test. Therefore it was worked out an adaptive filter on basis of the wavelet - transformation that on the first stage consistently deletes artifacts from EEG. The filter adapts itself to the location of the artifact in the window of analysis and to his form and asymmetry. After the filtration we built the wavelet - spectrums that are correlated with a corresponding functional test. We propose to start processing EEG in the largest scale, at that the lower frequencies consider like the drift of the isoline for higher. Sequentially subtracting the found frequencies from the EEG, we pass to research of high-frequency EEG components. This method is the consequence of more general theorem which is proved for any linear transformation.

#### S. B. GEMBARSKA $^1$

### Biharmonic and bianalytic functions having no tangential limits in a single circle

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Leading place in the boundary value problem of mathematical physics is occupied by researches of behavior of solutions to these tasks depending on properties of boundary conditions and a structure of limit of region. The question of the existence of the harmonic in the unit circle function that at any point does not have tangential limits, has been studied by Aikawa Hiroaki [1]. He proved that this function is harmonic Poisson integral.

Here we will construct bounded biharmonic and bianalytic functions having no tangential limits in the unit circle.

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#### V. GORODETSKYI $^1$ , M. OSADCHUK $^2$

## Identification of process model using experimental time series

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The problem of identification of the mathematical model often occurs in the study of systems of different physical nature. Let the estimated model (original system) be an autonomous system of ordinary differential equations with polynomial right-hand sides

$$\dot{x}_i = X_i(x_1, \dots, x_n),\tag{1}$$

where i = 1, ..., n. Then to find such a model, we can use the approach proposed in [1, 2]. If the experiment allows to obtain time series only for one observable variable, for example,  $x_1(t)$ , we can get relations connecting system (1) with a standard system [3]:

$$\dot{y}_1 = y_2, \quad \dot{y}_2 = y_3, \quad \dots, \quad \dot{y}_n = Y(y_1, \dots, y_n),$$
 (2)

where  $y_1(t) \equiv x_1(t)$ . After that, we will reconstruct numerically the system (2), and then using the connection between the systems (1) and (2), we can obtain a set of possible original systems (1).

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# Evolutional extensions and analogues of the operation of union for kinematic sets

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Theory of kinematic changeable sets [1] is based on the theory of changeable sets [2]. This theory may be used for simulation of the evolution of physical systems in the framework of different laws of kinematics.

In the present talk we introduce analogs of set-theoretic inclusion relation and set-theoretic operation of disjoint union for kinematic changeable sets and investigate their properties. The obtained results may be used for formulation of mathematical foundations of special relativity in the framework of theory of kinematic changeable sets.

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- [2] Ya.I. Grushka, arXiv:1207.3751v1, (2012).

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# Padé type approximants for some special power series of two variables

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By means of extension of V.K. Dzyadyk's method of generalized moment representation to case of two-dimensional number sequences [2] Padé type approximants are constructed and studied for functions of the form

$$f(z,w) = \frac{z^p}{z^p - w}\widetilde{f}(z) - \frac{w^{1/p}}{p} \sum_{r=0}^{p-1} \frac{\xi_r^{(p)} \widetilde{f}(w^{1/p} \xi_r^{(p)})}{z - w^{1/p} \xi_r^{(p)}},$$

where  $\widetilde{f}(z) = \frac{\Gamma(\nu+1)\Gamma(\sigma+1)}{\Gamma(\nu+\sigma+2)} \, {}_1F_1(1;\nu+\sigma+2;z)$ , a  ${}_1F_1(a;b;z)$  is a confluent hypergeometric function and  $\xi_r^{(p)} = e^{2\pi i r/p}, \ r = \overline{0,p-1}$ , are the p-th roots of unity.

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#### A. L. HULIANYTSKYI <sup>1</sup>

### Weak solvability and Galerkin discretization of a variable-order diffusion equation

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Let us consider the initial-value problem

$$\frac{\partial u(x,t)}{\partial t} - \Delta(K(x)D_0^{1-\alpha(x)}u(x,t)) = f(x,t), \tag{1}$$

$$u|_{t=0} = u_0(x), \qquad u|_{\partial\Omega} = 0, \tag{2}$$

where  $\Omega \subset \mathbb{R}^n$ ,  $n \in \mathbb{N}$ , is a bounded domain with a smooth boundary  $\partial \Omega$ ,  $D_0^{1-\alpha(x)}$  is the Riemann-Liouville fractional derivative of order  $1-\alpha(x)$  with respect to t with lower bound 0, and  $\Delta$  stands for the Laplace operator with respect to x.

Equations of the form (1) and their generalizations have recently been suggested as mathematical models of anomalous diffusion in inhomogenous media. They could possibly be applied to studying intracellular transport.

In the talk, weak solvabilty of (1)-(2) will be discussed. Also, a numerical method based on the space-time discretization of (1) will be presented.

#### P. S. JANCHUK <sup>1</sup>

# Quasispectral polynomials and their applications

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We build quasi-spectral polynomials in 80th years of the last century and used them to solve boundary value problems for differential equations; Cauchy problem for systems of ordinary nonlinear equations and other problems. For example, we have simple formulas of the Fourier quasispectral polynomial series for solutions Poisson equation with Dirichlet and Neumann boundary conditions. Based on the previously properties for quasispectral polynomials of the first and second kinds and associated Fourier series [1,2], we construct the algorithm to solving boundary value problem for the linearized Navier-Stockes equations

$$-\Delta v + qrad \ p = F, \quad div \ v = G$$

on a parallelepiped with the Dirichlet boundary conditions for v. We receive effective estimations of errors in  $L^2$  and uniform metrics.

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- [3] P.S. Janchuk, Questions of applied mathematics and mathematical modeling, Dnipropetrovsk (2012), p. 261–268.

#### I. M. KARABASH <sup>1</sup>

### When we can hear tones through a noise?

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The talk is devoted to the problem of recognition of an unknown and possibly infinite number of hidden frequencies in a discrete signal with a random noise. The talk is based on a joint research with Jürgen Prestin.

#### Yu. S. KOLOMOITSEV 1

### On approximation of periodic functions in Hölder and Sobolev spaces

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In the talk, we present some new results about approximation of  $2\pi$ -periodic functions by different polynomial methods in the Hölder spaces  $H_p^{r,\alpha}(0,2\pi)$  and the Sobolev spaces  $W_p^r(0,2\pi)$  for  $0 , and <math>0 < \alpha \leq r$ . We introduce several new objects for measuring the smoothness of functions and the errors of approximation in those spaces. The using of the introduced objects allows us to essentially improve the know results about approximation of functions in the mentioned spaces as well as to extent some results to the case 0 , which is not considered before.

This is a joint work with Jürgen Prestin (Universität zu Lübeck, Institut für Mathematik).

#### G. V. KRIUKOVA <sup>1</sup>

# RKHS-based linear functional strategy for machine learning

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In this talk a framework for discussing the generalization ability of a trained algorithms in the original function space using tools of functional analysis based on reproducing kernel Hilbert spaces (RKHS) is discussed. Using this framework, linear functional strategy approach is studied, and a new method for detecting relevant variables from a given high-dimensional data is developed. The effectiveness of the method is demonstrated in the example with synthetic data and in the reconstruction of real data.

This is a joint work with Sergei V. Pereverzyev (RICAM).

#### I. KUDYBYN <sup>1</sup>

## The use of vegetation to enhance erosion at the coast

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The paper is devoted to mathematical modeling of plant soil use for bank stabilization, which has low weight, high strength and flexibility. The root systems of bonded geomat ground that contribute to strengthening the plant layer.

#### A. KUKUSH $^1$

# Estimation of radiation risk in the presence of classical and Berkson errors in exposure doses

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Begin of body text We study the effect of measurement errors in exposure doses in a regression model with binary response. Recently it has been recognized that uncertainty in exposure dose is characterized by measurement errors of two types: (a) classical additive errors, and (b) Berkson multiplicative errors. In a simultion study based on data from radio-epidemiological research of thyroid cancer in Ukraine caused by Chornobyl accident, it is shown that ignoring measurement errors in doses leads to oversetimation of background prevalence and underestimation of excess relative risk. We propose several methods to reduce bias:

(a) new Regression Calibration, (b) SIMEX (simulation-extrapolation) that takes into account errors of both types, and (c) novel Corrected Score method.

The results are joint with Prof. I.A. Likhtarev, Dr. S.V. Masiuk, Dr. L.N. Kovgan (Radiation Protection Institute of ATSc of Ukraine), and Dr. S.V. Shklyar (Taras Shevchenko National University of Kyiv).

#### T. V. KUTIA <sup>1</sup>

# Computer simulation of heat, salt and moisture transfer influence on the stability of soil slopes

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Mathematical model of moisture transfer in the soil slope in terms of saturation function was improved taking into account the influence of heat and salts in liquid and solid phases of porous medium. Technological factors (heat and salt transfer) and fluid saturation of the porous medium were laid down in the basis of nonlinear dependencies of the soil strength characteristics [1]. The corresponding nonlinear boundary value problem was solved by collocation at points and radial basis function methods [2].

The resulting numeric data of fields distribution for saturation, temperature and concentration of chemicals were used for computer prediction of soil slope stability by determining the safety factor and slip curve position [3]. Numerical experiments and their analysis showed the dependence on the safety factor value and slip curve position of the investigated technological factors.

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- $[3]\,$  V. G. Fedorovsky, S. V. Kurillo,  $Geoecology~{\bf 6},~(1997),~p.~95–106.$

# Context-aware approach for processing heterogeneous data

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Multi-parameters system is an information system that support or organization decision-making activities. The problem in multi-parameters system is estimation multiple heterogeneous data in dynamic environment [1].

For this using context model, that can be used to characterize the situation of entities. We performed an analysis to determine a set of algorithms for context-processing tasks and to estimate their capability to fulfill the requirements typical for computing environments with different heterogeneous parameters that are using Multi-parameters system services. The results of context analysis can provide speeding up process of decision making.

 A. Kyselova, A. Shalaginov, CAD in Machinery Design. Implementation and Educational Problems:18-th UkranianPolish Conference CADMD2010 Lviv: Vezha Co (2010), p. 2022.

#### A. KYSELOVA 1, G. KYSELOV 2

# Tool for accumulation and classification of experimental anatomical data

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The most difficult task was using a neuroinformatics tools which should provide the filling of the gap between the obtained experimental data and semantic knowledge about them. Overcoming this gap should lead from achieving of data to the integrated understanding of the brain. From the scientific point of view in the artificial intelligence field, the most modern and promising direction is the field of semantic knowledge formalization, i.s. the ontological analysis of data. [1].

The purpose is to develop an application with tools for accumulation and classification of experimental anatomical and biological data. The main goal of this was to find a software solution capable of storing the procedures and the experiment descriptions in more organized way then just a file system.

Using the experience from the previous attempts and doing some research it became clear that the standard relational database approach will not work (or will be very cumbersome). The solution we propose is semantic wiki. Semantic extension gives us many extra features like database-like requests, external web API to the wiki and many others. Together with that it is still a wiki - users can create/delete/modify the pages the way they want. The main part of semantic wiki is Ontology. The ontology of a database is the definition of the elements and the relationships between the elements included in the database.

It is difficult to arrive at a standard ontology but important to strive towards clarification of definitions and concepts, allowing data to be more easily compared and interpreted.

[1] A. Kyselova, Visnyk NTUU KPI.Informatics, operation and computer science 58, (2013), p. 46–50.

O. KYSELOVA <sup>1</sup>, A. O. MATVIICHUK <sup>1</sup>

# Physiological signals automated analysis system based on virtual instruments

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The main aim of the research is to increase the informativities of the cardiovascular system automated analysis and to develop the virtual laboratory for support education in the field of biomedical engineering.

The research has proceeded in two directions: (1) cardiovascular system automated analysis based on Virtual Instruments; (2) remote physiological signal acquisition in the framework of E-learning for education in the field of biomedical engineering.

The research direction (1) is concerned with processing of signals collected of the remote ECG and 24-hours heart rate signals based on Virtual Instruments. For software development we are using NI LabVIEW software.

The research direction (2) is concerned with development the Elearning virtual laboratory with remote physiological signal acquisition.

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#### T. V. LOMAKO <sup>1</sup>

# On approximation of functions by algebraic polynomials in Hölder spaces

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We obtain new estimates of approximation of functions by algebraic polynomials in the Hölder spaces generated by the generalized Jacobi translations and by the Ditzian-Totik moduli of smoothness. By using modifications of the classical moduli of smoothness, we give improvements of the direct and inverse theorems of approximation and derive strong converse inequalities for some methods of approximation of functions in the Hölder spaces. As an example, we consider approximation by the Durrmeyer-Bernstein polynomial operators.

This is a joint work with Yurii Kolomoitsev (Institute of Mathematics NAS of Ukraine, Kiev) and Jürgen Prestin (Universität zu Lübeck, Institut für Mathematik).

#### T. MAHNO <sup>1</sup>

### Evolutionary approach to enhancement of medical ultrasound image processing

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It is considered the problem of improving ultrasound medical images processing using genetic algorithms. In this paper the algorithm is developed which allows to conduct a preliminary analysis of the image and the selection of appropriate processing methods. We performed an analysis to determine if there is an association between filters sequences and images texture parameters [1]. Ultrasound images of the human carotid artery at B-mode are used as the training and testing set. The set of filters and their optimal sequences are designed in previous work and used in current work.

The results of this analysis will help to improve image quality and detection of pathologies.

[1] T. Mahno, Electrotechnic and computer systems 18(94), (2015), p. 92 – 99.

O. S. MAKARENKO <sup>1</sup>, V. N. OSAULENKO <sup>2</sup>, A. N. DIDKIVSKYI <sup>2</sup>

### Some results and research problems in mathematical modelling of brain mechanisms: hierarchy structure, complex dynamics and cellular automata

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In the proposed talk the results of research for brain processes and mental nature diseases these problems are considered from the point of view of systems analysis and mathematical modelling. Artificial neural networks, cellular automata, differential and difference equations has been proposed.

The possibilities of design of algorithms for considering such phenomena are described. Some possibilities of the practical use of such algorithms are discussed. Specific features are exposed, on explaining and use of which in the algorithms of prognozing it is necessary to turn the special attention. Among them it is necessary to select high-frequency oscillations and micro-events in cells (neurons) for sudden death phenomena. The systems analysis of architecture, dynamics and desirable structure of models are conducted for processes at different levels of hierarchy into the brain. The hierarchical organization of brain architecture and corresponding hierarchical models are considered.

A research technique for brain neuron dynamics study with cellular automata identification methods is proposed. Possible approaches to identification with the usage of experimental data are described. Cellular automaton based on the neuron interaction principals is modelled. Concepts of information flows and consciousness are analyzed.

#### S. MAKSYMENKO <sup>1</sup>

### Notion of stability and its applications

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Stability of any kind of states or processes (states depending on time), such as planetary motion, work of mechanical/electronic devices, body health, etc., plays a principal role for in our life.

In general, stability of a certain state s of some system means that all states of this system closed to s have similar properties, whatever this means in a concrete situation. Therefore one should say about stability of sets of states. Then a set X of states might be called stable if it has the following property: if a state s belongs to s, then any state s sufficiently close to s also belongs to s.

One sees that such a definition resembles the definition of an open set in a topological space.

Therefore usually one defines a topology on the set of all states of a given system and then open sets of this topology serve as models for stable sets of states. The aim of this educational lecture is to review several topological results concerning stability, and describe their applications in life sciences.

#### P. M. MARTYNYUK $^1$

# About existence and uniqueness of a solution of a free boundary-value problem for a system of quasilinear parabolic equations

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We study a mathematical model of the process of filtration consolidation of soils under conditions of the influence of technogenic factors [1, 1, 3, 2], which is described with a boundary-value problem with a free boundary for a system of quasilinear parabolic equations. An existence and a uniqueness of a solution locally in time of the one-dimensional boundary-value problem in Hölder spaces are proved [5].

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#### O. V. MARUNKEVYCH <sup>1</sup>, S. I. MAKSYMENKO <sup>2</sup>

### Topological stability of averagings

 Data averaging is one of the principal instruments in many branches of science. In particular, averaging is used in computer graphics and digitizing of analog signals. One can observe that for not highly oscillating continuous signals the averaging "preserves" the forms of the initial signal provided that the averaging interval is sufficiently small. More thoroughly this means that these signals are topologically equivalent.

In our talk we will present sufficient conditions for a signal to be topologically stable with respect to small averagings.

We will also discuss the applications of the obtained results to computation of permutation and Kolmogorov-Sinai entropies.

S. V. MASIUK <sup>1</sup>

## Statistical methods in radiation epidemiology

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As a result of the 1986 Chornobyl accident, significant territory of Ukraine was under radioactive contamination and the inhabitants of that territory suffered from radioactive exposure. Even 5-6 years after the accident, an inflation of the incidence of thyroid cancer cases was observed for children and adolescents, who lived in the territories where the estimated thyroid exposure doses were quite high, see [1]. In fact, the growth of thyroid cancer prevalence for children and adolescents caused by internal irradiation from Chornobyl fallouts turned out to be the main (if not the unique) statistically reliable effect of the Chornobyl accident. Consequently this effect was of great interest for radiation epidemiologists all over the world, leading to a series of studies in Ukraine. However, interpretation of the results for most of the radiation epidemiological studies was based on risk estimation methods which did not take into account the presence of significant uncertainties in doses. One of the consequences of the assumption about the absence of errors in doses can be that the risk estimates are biased and the dose-response curve is distorted. The reasons for risk estimates distortions are not only systematic, but also due to random errors in the dose estimates, see [2]. The aim of present work is the analysis of the methods of radiation risk estimation in models with measurement errors in exposure doses.

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#### V. V. MYRONYUK <sup>1</sup>

### Trigonometric approximations and Kolmogorov widths of anisotropic Besov classes of periodic functions of several variables

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We obtained the exact order estimates for the approximations of periodic functions of several variables from the anisotropic Besov classes  $\mathbb{B}_{p,\theta}^{R}$  by means of trigonometric polynomials in the spaces  $L_q(\pi_d)$  (see [1]). The spectrum of trigonometric polynomials realizing approximation is contained in d-dimensional parallelepiped.

We also studied the behavior of Kolmogorov widths of the classes  $\mathbb{B}_{p,\theta}^{R}$ . It turns out that in some cases the subspace of trigonometric polynomials with spectrum in d-dimensional parallelepiped is the extremal subspace for approximation of the classes  $\mathbb{B}_{p,\theta}^{R}$  (see [1]).

[1] V. V. Myronyuk, Ukrainian Mathematical Journal 66 (8), (2015), p. 1248– 1266.

#### I. NESTERUK $^1$

### Can the low drag shapes of aquatic animals be applied in technology? Testing a special shaped body of revolution

<sup>1</sup>Institute of Hydromechanics NASU, Kyiv, Ukraine E-mail: inesteruk@yahoo.com The best swimmers have a streamlined shape that ensures an attached flow pattern even in the case of inertial motion (without varying the body shape). Similar rigid bodies of revolution were calculated and tested in the wind tunnel. The pressure drag measurements and the flow visualization on the model surface are presented.

V. OVERKO <sup>1</sup>, A. REDAELLI <sup>2</sup>

# Creation natural-like geometry of the human's aorta

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A 3-D geometry of human agrta and a mesh were constructed by using a bottom-up approach (in contrast to the top-down approach). The bottom-up approach means that you will first create some vertices, connect the vertices to create edges, and connect the edges to make; after this stitch the faces together to create volumes. While this process by its very nature requires more steps, the result is a valid geometry that can be used to generate the mesh. The mesh created in this part of work is intended for use in FLUENT. In order to meet this criterion, certain additional steps must be performed in GAMBIT. After creating the vertexes and curves that comprise the geometry, three faces are created: for input section, for output section and side surfaces. However, the Gambit do not have possibilities for creating well-fitting NURBS. In order to create nature-like forms of curves the MATLAB software is used. The algorithm and the code have been created. Feature of this software's combination is using the MATLAB's method of data approximation and GAMBIT's possibilities [1] - [3].

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- [2] B. Hahn. D.T. Valentine, Essential MATLAB for Engineers and Scientists, McGraw-Hill, Inc., Third edition, 2007.
- [3] S. Webb, The Physics of Medical Imaging, Institute of Physics Publishing, London.

#### O. PANASIUK <sup>1</sup>

# Regularized ranking by a linear functional strategy

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In this presentation we explore the possibility to employ regularization schemes for performing ranking tasks. We discuss a rather general approach to the regularization in reproducing kernel Hilbert space (RKHS) with the idea to mimic the best linear combination of the calculated regularized rankers corresponding to different values of the regularization parameter. The main result is the choice of the coefficients of the above mentioned linear combination that is based on the so-called linear functional strategy. We also discuss several applications where the proposed ranking algorithm can be effectively used.

This is joint work with G. Kriukova, S. Pereverzyev, P. Tkachenko.

#### R. PATIMAR <sup>1</sup>

# Mathematical models and fisheries management: Implications for effectiveness of models

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Mathematical models play major roles in fisheries management and ecological effect of fishery activities. In this paper, numerous stock assessments have been considered. The main question of interest is: how far fishery models are successful and efficient. Analysis shows that in recent years, the collapse of various important fish stocks has caused some critics to suggest that mathematical models actually obscure the truth by narrowing scientific understanding to the realm of quantifiable

events. Even though there is no satisfactory model, but models are necessary when evaluating the fish resources. So, it is in fact necessary to continue developing models using all possible ways of building models demanding respect for reality, not just mathematical elegance.

F. PIATTI <sup>1</sup>, S. PIROLA <sup>1</sup>, F. STURLA <sup>1</sup>, I. NESTERUK <sup>2</sup>, A. DELLA CORTE <sup>3</sup>, M. BISSELL <sup>4</sup>, F. PLUCHINOTTA <sup>5</sup>, M. LOMBARDI <sup>5</sup>, A. REDAELLI <sup>1</sup>

### 4D flow MRI: a clinical oriented platform for the evaluation of cardiovascular pathologies

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4D flow MRI is an established technique able to acquire information of the complex in-vivo fluid-dynamic phenomena during a cardiac cycle. This technique inherently offers the possibility to identify sound fluid-dynamic markers for a prompt evaluation of several cardiovascular pathologies [1]. However, 4D flow is not currently used in clinical practice due to its laborious post-processing. The proposed work focused on the application of a fast, easy-to-use and clinical-oriented Matlab-based platform for the analysis of 4D flow datasets.

To exploit the capabilities of the developed platform, we analyzed a heterogeneous pool of patients: an aortic coarctation (AC, age: 13; sex: Male), a congenital Fontan (CF, age: 20; sex: Female), an ischemic left ventricle (ILV, age: 63; sex: Male). A healthy volunteer (HV, age: 26; sex: Male) was analysed as a control. 4D flow MRI acquisitions were performed on a 1.5 T Siemens. The acquisition parameters were chosen accordingly to the anatomical district and pathology (e.g., isotropic voxels spacing of 1.7 - 2.3 mm3 and VENCs of 100-300 cm/s). The platform was adopted to investigate the effects of the different pathological fluid dynamics with respect to healthy conditions, in terms of flow rate

waveforms, velocity peaks, jets eccentricity, vorticity and helicity fields, and wall shear stress profiles [2].

The comparison between patient datasets and the healthy control conditions highlighted significant differences, by means of fluid dynamics alterations. Specifically, high velocity and wall shear stress peaks (e.g. AC patient in the descending aorta), dissipative vortex formations (e.g. ILV patient during the diastolic filling phase) and asymmetries of blood venous return (CF patient) were observed.

The analysis of the heterogeneous pool of patients was able to prove the capabilities of the developed platform. Significant differences were found between the patients and the control subject, by means of both qualitative and quantitative data. In conclusion, our aim was to provide a reliable and comprehensive support for clinicians , in order to deepen the analysis of cardiovascular pathologies relying on the in-vivo non-invasive measurement of the 4D blood hemodynamics.

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#### O.O. POKUTNYI <sup>1</sup>

## Turing bifurcation for boundary value chemical problems

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The report is devoted to obtaining the necessary and sufficient conditions for the existence of solutions of the following boundary value problem

$$\frac{\partial x}{\partial t} = (D_x \frac{\partial^2}{\partial r^2} - 1)x, \frac{\partial y}{\partial t} = D_y \frac{\partial^2 y}{\partial r^2} - \varepsilon x^2 y \tag{1}$$

with boundary conditions

$$\ell_1 x(\cdot) = \alpha_1, \ell_2 y(\cdot) = \alpha_2, \tag{2}$$

where  $\ell_1, \ell_2$  are linear operators.

Such model is a modification of Turing bifurcation model [1, p.122-124]. The most important critical periodic case will be considered. Equation for generating constant will be constructed.

Considering boundary value problem can be rewrite in the following operator form

$$\frac{dz}{dt} = Az + \varepsilon R(z),\tag{3}$$

$$\ell z(\cdot) = \alpha,\tag{4}$$

where  $z = (x, y)^T$ ,  $\alpha = (\alpha_1, \alpha_2)^T$ ,  $\ell = (\ell_1, \ell_2)^T$ .

The main results are applications of the obtained by authors results in [2]. Resonance case is connected with chaotic behavior [1, p.58]. Simple dynamic movement is violated by resonances.

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- [2] O.O. Pokutnyi, Journal of Mathematical Sciences 205(6), (2015), p. 821–831.

#### V. S. ROMANYUK <sup>1</sup>

### Nonlinear approximation of the Nikol'skii-Besov classes of the functions defined on the unit cube in the Euclidean space

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We represent definitions and properties of the multiple Haar basis system  $\mathbf{H}^d$  of functions defined on the unit cube  $\mathbb{I}^d$  in the Euclidean space  $\mathbb{R}^d$ ,  $d \geq 2$ .

We construct solution of the problem concerning exact order estimates of the best m-term approximation in the basis  $H^d$  of the Nikol'skii–Besov classes in the Lebesque spaces  $L_q(\mathbb{I}^d)$ ;

Algorithm of the construction of the extremal (in the sense of exact order estimates of the approximation) non-linear m-terms aggregate in the basis  $\mathbf{H}^d$  is given.

#### A. G. RUDNITSKII <sup>1</sup>

# Heart sound cancellation from lung sound recordings using empirical mode decomposition technique

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Auscultation is a noninvasive, low-cost and accurate method for assessing heart and respiratory diseases. The importance of this method is rezalt off high association of the acoustic signals generated by the lungs and the heart with the relevant pulmonary and cardial pathology. However, the importance of classic auscultation has decreased due to its inherent restrictions: physical limitations of human ear and the subjectivity of the examiner. This technique got the second breath when it became possible to add objectivity into auscultation data gathering by using computer complexes. However, the nonstationary character of the breathing noise and heart sounds make their identification a difficult task for the physician to perform. So, it is highly desirable can separate different components of the signal, suppress the noise and make the signal as audible as possible.

In this paper, it is proposed a new hybrid method based on *empirical mode decomposition* (EMD) technique [1] and suppression of noise in signal using *spectral subtraction technique* [1] for separation of heart sound signals from respiratory sound signals. The mixed signal is split into several components. Experiments have been conducted on simulated and real-life recorded mixed signals.

Proposed method needs minimum a priory information. In addition, EMD technique uses basis functions which are generated from signal and adaptive in nature. The proposed method has the ability to remove efficiently the heart sounds interference from lung sounds signals without any requirement of the reference signal unlike. This makes proposed approach more effective then other ones.

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### A Homogenized Problem for a Blood Transport Through Blood Vessels

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We study the mathematical model for blood flow through vessels, which can predict deviations of geometric structure of vessels from their initial healthy state (such as aneurysms and atherosclerosis). Aneurysms may induce turbulent regime of blood flow with extra friction, which in its turn may cause the further growth of aneurysms, thus increasing the risk of rupture. A ruptured aneurysm may lead to death.

In order to predict the behavior of aneurysms, we have to compute blood flow on a large time interval. The obstacle here is pulsatility of blood flow, which significantly increases time for computing the solution of this problem. Thus, asymptotical analysis has to be applied here.

For simplicity we consider a parabolic boundary-value problem. We construct an asymptotic approximation for the solution as the time interval infinitely increases.

### G. V. SANDRAKOV <sup>1</sup>

# Homogenization of hydrodynamics problems

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Turbulent regimes are arisen under a small viscosity and are associated with rapidly oscillating fluid dynamics. Moreover, in numerical modeling it is known that rapidly oscillation effects arise under computer simulations of solutions to Navier-Stokes equations with a vanishing viscosity. But reasons of the effects are not clear, since the effects may be

turbulent regimes or the numerical simulations may be incorrect. Some theoretical results in the direction will be presented in the report.

Homogenization of nonstationary Navier-Stokes equations with periodic rapidly oscillating initial data and the vanishing viscosity will be discussed. We give homogenized (limit) equations whose solutions determine approximations (asymptotic leading terms) of solutions to the equations under consideration and estimate the accuracy of the approximations. These approximations and estimates shed light on the following interesting property of the solutions of the equations. When the viscosity is not too small, the approximations contain no rapidly oscillating terms, and the equations under consideration asymptotically smooth the rapid oscillations of the data; thus, the equations are asymptotically parabolic. If the viscosity is very small, the approximations can contain rapidly oscillating terms with zero means, and the equations are hyperbolic.

Asymptotic and homogenizing methods are used for the consideration according to [1]. The results are applicable to some Kolmogorov flows.

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V. SEMENOV 1, 2

# Some properties of orthogonal wavelets based on Jacobi polynomials

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In this study we investigate wavelets based on Jacobi polynomials. We use the definition of n-th order wavelet as in [1]:

$$\psi_{n,r}(x) = \sum_{k=n+1}^{2n} P_k(t_r) P_k(x)$$
 (1)

for some fixed set of parameters  $t_1, t_2, ..., t_n$ . The definition of n-th order Jacobi polynomial depending on the parameters  $\alpha$ ,  $\beta$  is as follows

$$P_n^{(\alpha,\beta)}(x) = \frac{\Gamma(\alpha+n+1)}{n!\Gamma(\alpha+\beta+n+1)} \sum_{m=0}^n \binom{n}{m} \frac{\Gamma(\alpha+\beta+n+m+1)}{\Gamma(\alpha+m+1)}$$

$$\times \left(\frac{x-1}{2}\right)^m$$

where  $\Gamma()$  is a gamma function.

In this investigation we consider the following questions regarding wavelets (1):

- 1. Linear dependence. For which  $t_r$ , r=1,...,n, the wavelets  $\psi_{n,r}(x)$  are linearly dependent?
- 2. Orthogonality. For which  $t_r$ , r = 1, ..., n,  $\psi_{n,r}(x)$  are orthogonal?
- 3. Riesz stability. For which  $t_r$ , r = 1, ..., n, the minimum Riesz stability constant is achieved?

This investigation is a joint work with Professor Juergen Prestin (Universität zu Lübeck, Institut für Mathematik, Lübeck, Germany).

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E. V. SEMENOVA <sup>1</sup>, S. G. SOLODKY <sup>1</sup>

## Informational efforts for solving exponentially ill-posed problems

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We consider exponentially ill-posed problems in the spaces  $L_2(0,1)$  that are given as

$$Ax(t) \equiv \int_0^1 a(t,\tau)x(\tau)d\tau = f, \quad t \in [0,1].$$
 (1)

Here it is assumed that input data a and f are given with perturbation and function a has mixed partial derivatives up to order r by each variables. For numerical solving (1) with minimal informational efforts we proposed classical regularization methods with modified projection discretization (see [1]). According to our approach for solving (1) the regularization parameter is chosen by a balancing principle. It is proved that such numerical method is optimal by the order and saves informational efforts on minimal level.

### A. S. SERDYUK <sup>1</sup>, T. A. STEPANIUK <sup>2</sup>

# Order estimates of the best m-term trigonometric approximations of classes of analytic functions

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Let  $C_{\beta,p}^{\psi}$ ,  $1 \leq p \leq \infty$ , be the class of  $2\pi$ -periodic functions f, represented by the convolutions  $f(x) = \frac{a_0}{2} + \frac{1}{\pi} \int_{-\pi}^{\pi} \Psi_{\beta}(x-t) \varphi(t) dt$ ,  $\varphi \perp 1$ ,

$$\|\varphi\|_p \le 1$$
,  $a_0 \in \mathbb{R}$ , where  $\Psi_{\beta}(t) = \sum_{k=1}^{\infty} \psi(k) \cos\left(kt - \frac{\beta\pi}{2}\right)$ ,  $\sum_{k=1}^{\infty} \psi(k) < \infty$ ,  $\psi(k) > 0$ ,  $\beta \in \mathbb{R}$ .

Further, let us consider the quantities

$$e_m(C_{\beta,p}^{\psi})_s = \sup_{f \in C_{\beta,p}^{\psi}} \inf_{\gamma_m} \inf_{c_k \in \mathbb{C}} \|f(x) - \sum_{k \in \gamma_m} c_k e^{ikx} \|_s, \quad 1 \le p, s \le \infty,$$

where  $\gamma_m$ ,  $m \in \mathbb{N}$ , is an arbitrary set of m integer numbers.

We find that if sequence  $\psi(k)$  decreases to zero not slower than geometric progression, the following estimates are true:  $e_m(C^{\psi}_{\beta,p})_s \approx \psi\left(\left[\frac{m+1}{2}\right]\right)$ , where [a] is the integer part of a real number a.

### $N. SHVAI^{1}$

# Unitary equivalence criterion for unicellular operators on a separable Hilbert space

<sup>1</sup> National University of "Kyiv-Mohyla Academy" E-mail: nadiia.shvai@gmail.com In [1] and [2], we obtained the necessary and sufficient conditions that classify unicellular matrices up to unitary similarity. We extend this result to operators on a separable Hilbert space and apply it to Volterra operator problem.

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#### A. L. SHIDLICH <sup>1</sup>, S. O. CHAICHENKO <sup>2</sup>

### Some extremal problems in Orlicz spaces

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Let  $(\mathbb{R}^m, d\mu)$ ,  $m \geq 1$ , be the m-dimensional Euclidean space of points  $\mathbf{x} = (x_1, \dots, x_m)$  equipped with finite  $\sigma$ -additive continuous measure  $d\mu$ , let  $\mathbb{A}$  be a  $\mu$ -measurable subset of  $(\mathbb{R}^m, d\mu)$  whose  $\mu$ -measure is equal to  $a \in (0, +\infty]$ . An Orlicz function M(t),  $t \geq 0$ , is a non-decreasing convex down function satisfying the conditions M(0) = 0 and  $M(t) \to +\infty$  as  $t \to +\infty$ . Denote by  $L_M := L_M(\mathbb{A}; d\mu)$  the set of all functions f defined on  $\mathbb{A}$ , measurable with respect to the measure  $d\mu$ , such that  $(\forall C > 0) : \int_{\mathbb{A}} M(C|f(\mathbf{x})|) d\mu < +\infty$ . Equipped with the norm  $\|f\|_{L_M} := \inf\{\alpha > 0 : \int_{\mathbb{A}} M(|f(\mathbf{x})|/\alpha) d\mu \leq 1\}$  the linear space  $L_M$  is the Banach space and called Orlicz space. Also we set  $L_p := L_M$ , when  $M(t) = t^p$ ,  $0 , and denote by <math>U_p^+(\mathbb{A})$  the subset of all nonnegative functions from unit ball of  $L_p$ ,  $\mathcal{U}_p := U_p^+ \cap L_M$ .

Further, let  $f \in L_M$  and  $\sigma > 0$ . Consider the quantities

$$e_{\sigma}(f)_{L_M} := \inf\{\|f - \chi_{\gamma_{\sigma}}f\|_{L_M} : \operatorname{mes}_{\mu}\gamma_{\sigma} = \sigma\},$$

where  $\chi_{\gamma_{\sigma}} = \chi_{\gamma_{\sigma}}(\mathbf{x})$  is the characteristic function of the set  $\gamma_{\sigma}$  ( $\chi_{\gamma_{\sigma}}(\mathbf{x})=1$ , when  $\mathbf{x} \in \gamma_{\sigma}$  and  $\chi_{\gamma_{\sigma}}(\mathbf{x})=0$ , when  $\mathbf{x} \notin \gamma_{\sigma}$ ).

**Theorem.** Suppose that  $p \in (0, \infty)$ ,  $\varphi$  is a nonnegative essentially bounded on  $\mathbb{A}$  function that  $\lim_{|\mathbf{x}| \to +\infty} \varphi(\mathbf{x}) = 0$ , when  $\max_{\mu} \mathbb{A} = +\infty$ ,

and M is Orlicz function such that the function  $M(t^{1/p})$  also is Orlicz function. Then for any  $\sigma \in (0, a)$ ,

$$e_{\sigma}(\varphi,p)_{L_M} \coloneqq \sup_{y \in \mathcal{U}_p} e_{\sigma}(\varphi\,y)_{L_M} = \sup_{s \in (\sigma,a]} \Big( \int_0^s \bar{\varphi}^{-p}(t)dt \Big)^{-\frac{1}{p}} \Big( M^{-1}(\frac{1}{s-\sigma}) \Big)^{-1},$$

where  $M^{-1}$  is the converse function of the function M,  $\bar{\varphi}(t)$  is the decreasing rearrangement of  $\varphi(\mathbf{x})$ . The least upper bound on the right-hand side of the last relation is realized for a certain finite value  $s = s^*$ .

Note that in the case, where  $M(t) = t^q$ , for all  $0 < q < \infty$ , similar assertions were obtained by A.I. Stepanets and A.L. Shidlich.

S. V. SIRYK <sup>1</sup>, N. N. SALNIKOV <sup>2</sup>

### A choice of weight functions in the Petrov-Galerkin FEM for solving convection-diffusion problems

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The finite-element Petrov-Galerkin method (PGM) is one of the most successful approaches to the construction of numerical approximations for differential equations. The key role of PGM applying for solving convection-dominated is the correct choice of the weighting functions which prevents spurious oscillations in the numerical solutions and stabilizes the numerical solution while maintaining acceptable accuracy. The option of the weighting functions selection for the integration of one-, two- and three-dimensional convection-diffusion problems was proposed in [1, 2, 3, 4, 5]. These weight functions were later successfully applied for the numerical solution of various unsteady convection-diffusion-reaction problems (both linear and nonlinear) [1, 3, 5, 6]. Some new estimates of PGM for steady 1D convection-diffusion equations were obtained in [7] and later were generalized for problems with reaction terms. Various

aspects of mass lumping technology were considered in [8, 9]. The current report presents an overview and generalization of some results of the papers mentioned above.

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#### A. V. SOLODUN <sup>1</sup>

## Nonlinear liquid sloshing in a truncated conical tank

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The studies of nonlinear resonance liquid sloshing of an ideal incompressible liquid in a rigid truncated conical tank are presented in the report. The tank performs small-magnitude oscillatory motions with the forcing frequency close to the lowest natural sloshing frequency. The multimodal method combined with nonconformal mapping technique and the MoiseevNarimanov asymptotic relationships [1, 2] are employed to derive a infinite-dimensional modal system of nonlinear modal equations modeling the resonant slosh dynamics. Derived modal system allows us to analyze a steady-state regimes and its stability. Utilizing the Lukovsky's formula [1] makes it possible to evaluate liquid force response when tank is under external harmonic excitation. Experimental researches allow us to determine safe exploitation regimes of complex structures with the conical cavities.

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#### Yu. Yu. SOROKA 1

### Non-singular foliations in the plane

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Control theory is the area of applied mathematics which researches the control of certain physical processes and systems. This theory has connection with classical branches of mathematics. Some questions of it leads to the study of foliation theory as well.

In our report we will consider non-singular foliations in the plane and describe the structure of group of automorphisms on such foliation. As well we will present the application of non-singular foliations in the control theory.

### S. A. STASYUK $^1$

### Sparse trigonometric approximation of Nikol'skii-Besov classes of generalized mixed smoothness

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Investigations are devoted to finding the order estimates of the best m-term trigonometric approximation (it is a type of sparse approximation) of Nikol'skii-Besov classes of generalized mixed smoothness in an integral and a uniform metrics. Obtained estimates in an integral metric have the exact order. The mentioned investigations originate from the author's papers [1, 2]. The estimates from [1, 2] were not obtained by a constructive method. But now it is possible to obtain these estimates

with the constructive method. These estimates are achieved by a constructive greedy-type algorithm. Initially, such type of algorithm was developed by Temlyakov [3]. After that we have generalized and applied it to our problem.

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### I. STOLZ <sup>1</sup>, K. KELLER <sup>2</sup>

# Evaluating time series: an ordinal pattern approach

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The Kolmogorov-Sinai entropy is a commonly used measure of complexity of dynamical systems, its determination, however, is not easy. Bandt and Pompe [1] invented the concept of permutation entropy, which has been shown to be close to the Kolmogorov-Sinai entropy and to be an interesting tool for evaluating time series (see Bandt et al. [2], Amigó [3] and Keller et al. [4]). This measure utilizes so called ordinal patterns and their distribution to access information stored in the measurements of an underlying system. These ordinal patterns describe the up and down behavior of measurements, i.e. in this approach the rank order of consecutive values of a time series is considered instead of the values themselves. The question arises weather all relationships between the consecutive values have to be considered or weather, in order to reduce accessing the same information multiple times, the approach can be modified. The aim of this talk is to introduce shortly the ordinal pattern approach and to present modifications, consequences as well as applications with related experimental data.

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### F. STURLA $^1,$ A. CAIMI $^1,$ F. PIATTI $^1,$ K. MANNING $^2,$ A. REDAELLI $^1$

# Fluid-structure interaction modeling of a pediatric ventricular assist device

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Cardiovascular diseases are the main cause of death in the world and, due to the limited availability of fresh organs, Ventricular Assist Devices (VADs) are adopted as an effective bridge to transplantation. In particular, appropriate pediatric VADs are currently under development in order to reduce the thrombogenic potential of previous rescaled adult devices [1].

We developed a comprehensive and innovative characterization of a 12cc pneumatic pediatric Ventricular Assist Device (pVAD) adopting a Fluid-Structure Interaction (FSI) approach able to reliably reproduce the realistic behavior of both blood and air chambers, thus focusing on the dynamics of the thin membrane separating the two fluids. Comparable working conditions, between our computational model and an experimental mock circulatory loop, were achieved extracting in vitro experimental pressure waveforms; these were then applied, as boundary conditions to the blood and air domains. The explicit finite element solver LS-DYNA (Livermore, CA, USA) was adopted: simulations were performed using a penalty-coupling algorithm in order to couple the eulerian fluid elements to the lagrangian structural ones [2].

Computational velocities were comparable with the experimental ones, acquired by means of particle image velocimetry (PIV). Both the numerical inlet and outlet velocities were within the experimental range, with

a maximum velocity between 0.5 and 0.7 m/sec. The FSI derived kinematics of the membrane well compared, in terms of displacement and velocity, with the ones extracted from the mock-loop in vitro tests using an High Speed Video Camera.

The developed FSI model proved able to provide a comprehensive assessment of the continuous, time-dependent and threedimensional pVAD fluid dynamics as well as to capture the threedimensional and asymmetric kinematics of the pVAD membrane.

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### I. A. TERESHCHENKO <sup>1</sup>

### Methodology of Petrov-Galerkin weighting functions choice with usage of neural networks and group method of data handling for convection-diffusion problem

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Over the past four decades, the finite element method has become one of the most widely used methods for solving boundary value problems of mathematical physics. However, for problems with superior convection processes, classical approaches may give unstable solutions. To overcome this drawback is proposed to use the weighting function different from basic function (Petrov-Galerkin method). The disadvantage of such approaches is an empirical selection of the weight parameters that need to be reconstructed for each new task. The most Petrov-Galerkin formulations take into account the spatial discretization and the weighting functions developed give satisfactory solutions for steady state problems. Though these schemes can be used for transient problems, there is scope for improvement. Nowadays, research is being carried out for the selection of the weighting functions using neural networks and group method of data handling [1]. This approach allowed us to dynamically adapt the

weighting functions during the integration. Also were proposed a new way of determining the parameters of the SUPG weighting functions based on GMDH. Both of these approaches has shown high efficiency for dominant convection process. Proposed methods of adaptive selection of stabilizing parameters of the proposed weighting functions that take into account the evolution of the solution, the vector field of the convective flow and depending convection-diffusion flow. Reducing the amount of calculation of parameters of the weighting function, based on the behavior of the velocity field of the convective flow, significantly reduce the dimension of the problem.

 I. A. Tereshchenko International Journal of Innovative Technology and Research 2(5), (2014), p. 1237–1242.

### L. TERESHCHENKO $^1$

# Application of solutions to magneto-elasticity problem in medical diagnostics

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In the paper I consider an application of the magneto-elasticity methods for researching processes in medical diagnostics. It is allows us to create a new technologies of the treatment of various diseases people.

Soft magnetic alloys are easily magnetized and demagnetized. They are uniquely suited for specialized applications that require high permeability, low losses, and low residual magnetism. Magnetic materials can be applied to cell separation, immunoassay, magnetic resonance imaging (MRI), drug and gene delivery, minimally invasive surgery, radionuclide therapy, hyperthermia and artificial muscle applications. Physical properties which make magnetic materials attractive for biomedical applications are, first, that they can be manipulated by an external magnetic field. This feature is useful for separation, immunoassay and drug targeting. Second, hysteresis and other losses occur in alternating magnetic fields, this is useful in hyperthermia applications. Some magneto-tactic bacteria are known to respond to a magnetic field, they contain chains of

small magnetite particles and they can navigate to the surface or bottom of the pools that they live in using these particles.

Often soft magnetic materials are heterogeneity technological, structural or structural nature, such as oral, enthetic inclusion holes. Research on strength, reliability and durability of structural elements leads to the need to determine the stress and strain state and the induced magnetic field in the body and heterogeneity. Therefore, it was decided a number of problems on plane and in space for magnetically soft ferromagnetic materials by harmonic functions.

#### I. V. VERBYTSKYI <sup>1</sup>

# Nonlinear eigenvalues calculation for optimal resonances in one-dimensional optical cavities

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Optical cavity is device where the electromagnetic field is well confined. Volume of cavity is a critical parameter than the cavity structure optimization is urgent task [1].

Fundamentals of one-dimensional resonator optimization are described in [2]. The optimization procedure is calculation of cavity structure, when decay of electromagnetic field is minimal. In this case it is considered that cavity consists of layers with piecewise continuous dielectric permittivity in range  $\varepsilon_{\min}...\varepsilon_{\max}$ . For derive the optimal structure of cavity are necessary calculate quasi-normal (QN) eigenvalues of nonlinear complex function  $W(l,\omega,\theta)$ , where l is a cavity length,  $\omega=\alpha-i\beta$  is complex QN eigenvalue,  $\theta$  is an initial phase of the electromagnetic field. The width of layers is defined by solving equations  $Re(E(x_k,\omega,\theta))=0$ ,  $Im(E(x_k,\omega,\theta))=0$ , where E is the electric field intensity,  $x_k$  is the width of the layer number k.

Functions W, E are transcendental, so calculation of eigenvalues is very time consumption problem. For eliminating of calculations, procedure of localization of roots is developed. Also effective numerical methods for calculation of roots are proposed.

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## Hydroacoustics of the prosthetic bileaflet mitral valve

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The report presents the research results of jet flow noise of the open and semi-closed bileaflet mitral valve. The studies were conducted in microlaboratory of the Polytechnic Institute in Milan. Hydrodynamic noise was measured by the miniature absolute pressure and pressure fluctuation sensors inside the test bench with the prosthetic bileaflet mitral valve. Hydroacoustics central and side jets that flowed from open and semi-closed mitral valve was studied for four water discharge from the 5L/min to 20l/min. The changes integral and spectral characteristics of the field of pressure fluctuations, depending on the flow regime and the condition of the mitral valve were detected. It was found that for a constant water flow conditions inside the atrium (20 1/min), hydrodynamic noise inside the bench was increased nearly 5 times in the frequency range (55-65) Hz and nearly 2 times in the frequency range (10-150) Hz in the case of the semi-closed valve. Experimental researches have shown that the valve flow noise measurements can be an effective diagnostic tool of the artificial bileaflet mitral valve operation.

### Generation features of the coherent vortex structures by cavities and bumps on a streamlined surface

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Discontinuity streamlined surface in the form of cavities and bumps are widespread in nature and technology. In the conditions of turbulent flow they generate coherent vortex structures which significantly increase the mixing and mass transfer processes over streamlined surfaces. The large scale vortex structures, circulating and jet currents are formed inside the cavities [1]. They generate intensive fields of velocity, pressure, temperature, vorticity, and also periodically and quasiperiodically are ejected outside of cavities in boundary layer. Horseshoe and wake vortices are generated about bluff body when a boundary layer is separated from streamlined surface [2]. These vortex structures, under certain flow conditions, in the wake of the bumps form the Karman vortex street. The presence of coherent vortex structures in shear and separation flows leads to changes in the space-time characteristics of the flow. The wave and frequency spectra appear tonal components, which are caused by the characteristic features of the formation and evolution of vortex structures inside and around the streamlined surface discontinuities. Cross-correlation and spectral functions allow us to identify the scale, lifetime, direction and transfer velocity of the coherent vortex structures [1-3]. [1]

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#### C. WÜLKER<sup>1</sup>

### Towards a fast Fourier transform for spherical Gauss-Laguerre basis functions

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Spherical Gauss-Laguerre basis functions (i.e., normalized functions of the type  $\exp(-r^2/2)r^lL_{n-l-1}^{l+1/2}(r^2)Y_{lm}(\vartheta,\varphi)$  of order  $n \in \mathbb{N}$  and degrees  $l=0,...,n-1, |m| \leq l; Y_{lm}$  being the spherical harmonics) are used extensively e.g. in molecular dynamic simulations, as well as in computational quantum mechanics in general. However, to the present, there is no reliable algorithm available to compute the Fourier coefficients of a function with respect to spherical Gauss-Laguerre (SGL) basis functions in a fast way. In this work, we combine the results of [1] and [2] to derive an SGL sampling theorem that permits an exact computation of the SGL Fourier expansion of a given function, provided that the expansion is of order at most  $N \in \mathbb{N}$ . The fact that the SGL basis functions satisfy a three-term recurrence relation in the order n then allows us to use the techniques of [3] to state a fast SGL Fourier transform. This algorithm has complexity  $\mathcal{O}(N^3\log^2 N)$ , instead of the complexity  $\mathcal{O}(N^6)$ associated with the direct approach, and is currently investigated for its performance.

This is a joint work with J. Prestin, Institute of Mathematics, University of Lübeck, Lübeck, Germany.

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# Approximation of functions from the isotropic Nikol'skii–Besov classes

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In the present talk, we study some problems of the approximation of the isotropic Nikol'skii–Besov classes  $B_{p,\theta}^r(\mathbb{R}^d)$  [1] in the metrics of space  $L_{\infty}(\mathbb{R}^d)$  and  $L_1(\mathbb{R}^d)$ .

We obtained the exact order estimates for the approximations of Nikol'skii–Besov classes by sums of de la Vallee Poussin type [2, p. 358] with support of their Fourier transforms concentrated in d-dimensional parallelepiped.

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# Internal differential turning point of the second kind for singularly perturbed systems

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We construct uniform asymptotic solution for a singular perturbed (SP) system of the 4-th order with an internal differential turning point of the second kind. Previous studies of the scalar linear ordinary differential equation [1], play a fundamental role in the problem of the stability of

parallel flow of the viscous fluid and SP system of the 3-th order with an internal differential turning point of the first kind [2].

A uniform asymptotic solution of the system has been constructed with the method of essential singular functions and Airy-Dorodnitsyn model operator for the case of the stable turning point; asymptotic forms of the solutions of the system have been constructed on a segment that includes the turning point.

The main difficulty, in case of internal turning point, is construction of smooth solutions for a system of singular perturbed differential equations on a polygon including this turning point.

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### Microfluidic emulators of ventricular assist device shear stress patterns: Lab-on-a-Chip test benches for platelet activation under dynamic device-like shear flow

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Ventricular assist devices (VADs) are currently used to provide restoration of hemodynamics in patients with heart failure. However, VADs have been frequently affected by thrombosis, mainly due to the altered shear stress conditions they expose platelets to [1, 2]. VAD patients are treated with anti-thrombotic drugs, but these therapies often turned out to be ineffective. Many research have been conducted on this issue, which highlighted the need for frequently monitoring the thrombotic risk of patient blood under dynamic shear flow conditions and under the effect of

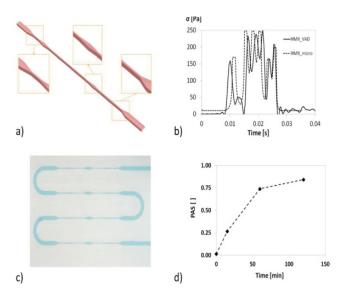


Figure 1: Figure 1. a) 3D CAD of the designed microfluidic channel. b) Significant shear stress waveforms of the microfluidic channel and the HMII VAD. c) Optical microscope image of the PDMS microfludic platform. d) PAS over time obtained from the in vitro tests.

different drugs, with the aim of allowing personalization of anti-platelet therapies and enhancing device performances and patient safety.

In this context, we designed and realized novel PDMS-based microfluidic platforms able to replicate significant shear stress patterns of medical devices, with the aim of developing a Lab-on-a-Chip device that can be used as a test bench for platelet activation studies under VAD-like shear stress flow conditions. As a test case, we considered the HeartMateII VAD (HMII, Thoratec Corportation, Pleasanton, CA, USA), that was studied through computational fluid dynamic (CFD) analyses in a previous work [3], in which significant dynamic shear stress waveforms along platelet trajectories were extracted.

By means of CFD analyses, we designed a microfluidic channel able to replicate one of those waveforms when perfused at a constant flow rate. First prototypes of PDMS microfluidic platforms were realized through standard soft lithography, starting from the designed channel unit. Preliminary in vitro tests were performed with the aim of characterizing the PDMS devices: the platforms were perfused in a closed loop configuration with ovine gel-filtered platelet (GFP) using a peristaltic pump. Platelet activation at different time points was tested using the platelet activity state (PAS) assay [4]. The design of the microfluidic channel unit (HMII microfluidic) is shown in Fig. 1a. The channel is characterized by a constant height (50 m), while its width was modulated longitudinally in order to generate the desired shear stress pattern. In Fig. 1b, a significant shear stress waveform of the HMII microfluidic is shown and compared to the HMII VAD waveform. The results of the in vitro tests performed on the PDMS microfluidic platforms (Fig. 1c) are reported in Fig. 1d, where the PAS is shown at different time points. The platforms induced a high level of platelet activation with a non linear trend, which tended to a plateau close to the maximum level of PAS of the sample. In this work we demonstrated the feasibility of using microfluidic technologies to design microfluidic platforms able to replicate the dynamic shear stress patterns of VADs. We performed preliminary characterization tests of the microfluidic devices. The approach seems promising to develop a novel technology that can be advanced to Pointof-Care devices for monitoring patient thrombogenic risk under realistic shear flow conditions. Current studies are focused on developing and testing an on-chip platelet activation assay, with the aim of downscaling and integrating the reading onto the microfluidic platform and advance the approach towards a potential Point-of-Care testing procedure.

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